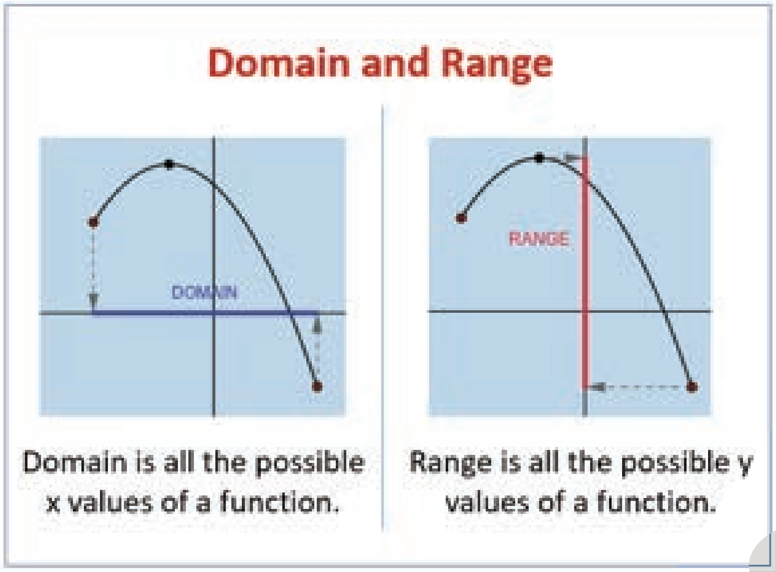
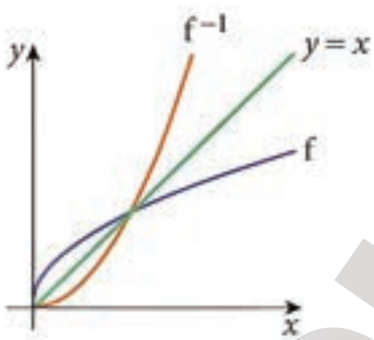
Functions



- For any one-to-one function f(x), there is an inverse function $f^{-1}(x)$.
  The curves of a function and its inverse are reflections of each other in the line y = x.
- f^{-1} exists only if the function is one-to one



- Domain of $f^{-1}(x)$= Range of $f(x)$
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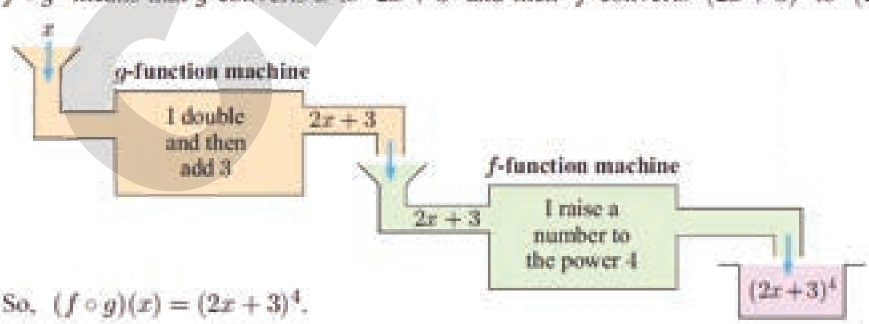
Composite function:

Consider $f: x \mapsto x^4$ and $g: x \mapsto 2x+3$.

$f \circ g$ means that g converts x to $2x+3$ and then f converts $(2x+3)$ to $(2x+3)^4$.



So, $(f \circ g)(x) = (2x+3)^4$.

Exercise 1:

1 If $f(x) = 3x - x^2 + 2$, find the value of:

a $f(0)$ **b** $f(3)$ **c** $f(-3)$ **d** $f(-7)$ **e** $f(\frac{3}{2})$

2 If $g : x \mapsto x - \frac{4}{x}$, find the value of:

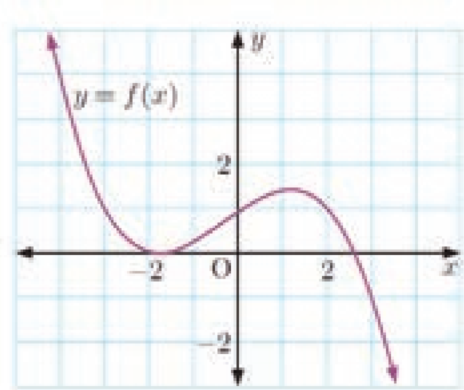
a $g(1)$ **b** $g(4)$ **c** $g(-1)$ **d** $g(-4)$ **e** $g(-\frac{1}{2})$

3 The graph of $y = f(x)$ is shown alongside.

a Find:

i $f(2)$ **ii** $f(3)$

b Find the value of x such that $f(x) = 4$.



4 If $f(x) = 7 - 3x$, find in simplest form:

a $f(a)$ **b** $f(-a)$ **c** $f(a+3)$ **d** $f(b-1)$ **e** $f(x+2)$ **f** $f(x+h)$

5 If $F(x) = 2x^2 + 3x - 1$, find in simplest form:

a $F(x+4)$ **b** $F(2-x)$ **c** $F(-x)$ **d** $F(x^2)$ **e** $F(x^2-1)$ **f** $F(x+h)$

6 Suppose $G(x) = \frac{2x+3}{x-4}$.

a Evaluate: **i** $G(2)$ **ii** $G(0)$ **iii** $G(-\frac{1}{2})$

b Find a value of x such that $G(x)$ does not exist.

c Find $G(x+2)$ in simplest form.

d Find x if $G(x) = -3$.

Coordinate Geometry

The gradient, *m* joining points (x_1, y_1) and (x_2, y_2) is $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

- When 2 lines parallel, $m_1 = m_2$
- When 2 lines perpendicular, $m_1 m_2 = -1$

The distance between 2 points= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of 2 points = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

- Equation of straight lines:

$y = mx + c$

$y - y_1 = m(x - x_1)$

$ax + by + c = 0$

1.

When e^{2y} is plotted against x^2, a straight line graph passing through the points (4, 7.96) and (2, 3.76) is obtained.

(a) Find y in terms of x. [5]

(b) Find y when $x = 1$. [2]

(c) Using your equation from **part (a)**, find the positive values of x for which the straight line exists. [3]

2.

Variables x and y are such that when $\sqrt{y}$ is plotted against $\log_2(x+1)$, where $x > -1$, a straight line is obtained which passes through $(2, 10.4)$ and $(4, 15.4)$.

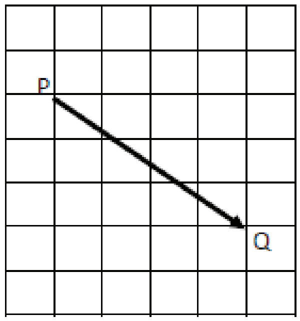
(a) Find $\sqrt{y}$ in terms of $\log_2(x+1)$. [4]

(b) Find the value of y when $x = 15$. [1]

(c) Find the value of x when $y = 25$. [3]

Vector

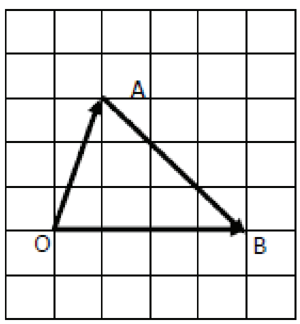
Vector notation and position vector



From the diagram, vector $\overrightarrow{PQ} = \binom{4}{-3}$ or $4\boldsymbol{i} - 3\boldsymbol{j}$

Modulus of vector $\overrightarrow{PQ}$, $|\overrightarrow{PQ}|$ means the magnitude or the length of the vector

$$|\overrightarrow{PQ}| = \sqrt{4^2 + (-3)^2} = 5$$



The position vector of a point A relative to origin means the displacement of the point A from O

$\overrightarrow{OA} = \binom{1}{3}$ or $\boldsymbol{i} + 3\boldsymbol{j}$

$\overrightarrow{AB}$ means the position vector of Q relative to P

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AO} = -\overrightarrow{OA} + \overrightarrow{OB}$

Unit vector: vector of length 1 unit

Example, unit vector of $\overrightarrow{PQ} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{4i-3j}{5} = \frac{4}{5}\boldsymbol{i} - \frac{3}{5}\boldsymbol{j}$

Collinear points:

If $\overrightarrow{AB} = k\overrightarrow{AC}$, then ABC are collinear (points A,B and C are lying in the same straight line)

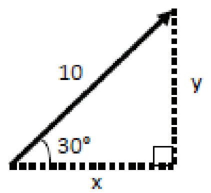
Constant velocity problems

Velocity is a quantity that has both magnitude and direction, $velocity = \frac{displacement}{time}$

Speed only has magnitude, $speed = \frac{distance}{time}$

Example:

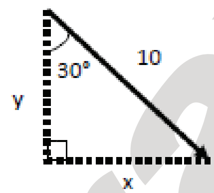
V=3$\boldsymbol{i}$+4$\boldsymbol{j}$ ms^{-1} Speed=$\sqrt{3^2+4^2} = 5ms^{-1}$



$x = 10\cos 30° = 5\sqrt{3}$

$y = 10\sin 30° = 5$

Velocity vector= $5\sqrt{3}\boldsymbol{i} + 5\boldsymbol{j}$



$x = 10\sin 30° = 5$

$y = 10\cos 30° = 5\sqrt{3}$
(negative y because moving downwards)

Velocity vector= $5\boldsymbol{i} - 5\sqrt{3}\boldsymbol{j}$

$r = a + tv$

Where r= final position vector

a= initial position vector

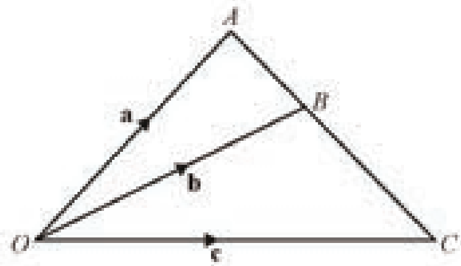
v= velocity vector

t= time

If object a and b collide, then $r_a = r_b$

1.

(a)



The diagram shows triangle OAC, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The point B lies on the line AC such that $AB:BC = m:n$, where m and n are constants.

(i) Write down $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Write down $\overrightarrow{BC}$ in terms of $\mathbf{b}$ and $\mathbf{c}$. [1]

(iii) Hence show that $n\mathbf{a} + m\mathbf{c} = (m+n)\mathbf{b}$. [2]

(b) Given that $\lambda\begin{pmatrix}2\\1\end{pmatrix} + (\mu - 1)\begin{pmatrix}-4\\7\end{pmatrix} = (\lambda + 1)\begin{pmatrix}4\\-2\end{pmatrix}$, find the value of each of the constants λ and μ. [4]