

**Functions**

(Past Year Topical Questions 2012-2017)

Oct/Nov 2012 (11)

9.

A function  $g$  is such that  $g(x) = \frac{1}{2x-1}$  for  $1 \leq x \leq 3$ .

(i) Find the range of  $g$ . [1]

(ii) Find  $g^{-1}(x)$ . [2]

(iii) Write down the domain of  $g^{-1}(x)$ . [1]

(iv) Solve  $g^2(x) = 3$ . [3]

Oct/Nov 2013 (11)

12.

(a) A function  $f$  is such that  $f(x) = 3x^2 - 1$  for  $-10 \leq x \leq 8$ .

(i) Find the range of  $f$ .

[3]

(ii) Write down a suitable domain for  $f$  for which  $f^{-1}$  exists.

[1]

(b) Functions  $g$  and  $h$  are defined by

$$g(x) = 4e^x - 2 \text{ for } x \in \mathbb{R},$$

$$h(x) = \ln 5x \text{ for } x > 0.$$

(i) Find  $g^{-1}(x)$ .

[2]

(ii) Solve  $gh(x) = 18$ .

[3]

Oct/Nov 2013 (13)

5.

For  $x \in \mathbb{R}$ , the functions  $f$  and  $g$  are defined by

$$f(x) = 2x^3,$$

$$g(x) = 4x - 5x^2.$$

(i) Express  $f^2\left(\frac{1}{2}\right)$  as a power of 2.

[2]

(ii) Find the values of  $x$  for which  $f$  and  $g$  are increasing at the same rate with respect to  $x$ . [4]

May/June 2015 (11)

8.

It is given that  $f(x) = 3e^{2x}$  for  $x \geq 0$ ,  
 $g(x) = (x + 2)^2 + 5$  for  $x \geq 0$ .

(i) Write down the range of  $f$  and of  $g$ . [2]

(ii) Find  $g^{-1}$ , stating its domain. [3]

(iii) Find the exact solution of  $gf(x) = 41$ . [4]

(iv) Evaluate  $f'(\ln 4)$ . [2]

Oct/Nov 2015 (11)

11.

(a) A function  $f$  is such that  $f(x) = x^2 + 6x + 4$  for  $x \geq 0$ .

(i) Show that  $x^2 + 6x + 4$  can be written in the form  $(x + a)^2 + b$ , where  $a$  and  $b$  are integers. [2]

(ii) Write down the range of  $f$ . [1]

(iii) Find  $f^{-1}$  and state its domain. [3]

(b) Functions  $g$  and  $h$  are such that, for  $x \in \mathbb{R}$ ,

$$g(x) = e^x \quad \text{and} \quad h(x) = 5x + 2.$$

Solve  $h^2g(x) = 37$ .

[4]

May/June 2016 (11)

6.

The function  $f$  is defined by  $f(x) = 2 - \sqrt{x+5}$  for  $-5 \leq x < 0$ .

(i) Write down the range of  $f$ . [2]

(ii) Find  $f^{-1}(x)$  and state its domain and range. [4]

The function  $g$  is defined by  $g(x) = \frac{4}{x}$  for  $-5 \leq x < -1$ .

(iii) Solve  $fg(x) = 0$ . [3]



May/June 2017 (11)

4.

(a) It is given that  $f(x) = 3e^{-4x} + 5$  for  $x \in \mathbb{R}$ .

(i) State the range of  $f$ .

[1]

(ii) Find  $f^{-1}$  and state its domain.

[4]

(b) It is given that  $g(x) = x^2 + 5$  and  $h(x) = \ln x$  for  $x > 0$ . Solve  $hg(x) = 2$ .

[3]

Oct/Nov 2017 (11)

6.

(a) Functions  $f$  and  $g$  are such that, for  $x \in \mathbb{R}$ ,

$$f(x) = x^2 + 3,$$

$$g(x) = 4x - 1.$$

(i) State the range of  $f$ .

[1]

(ii) Solve  $fg(x) = 4$ .

[3]

(b) A function  $h$  is such that  $h(x) = \frac{2x+1}{x-4}$  for  $x \in \mathbb{R}$ ,  $x \neq 4$ .

(i) Find  $h^{-1}(x)$  and state its range.

[4]

(ii) Find  $h^2(x)$ , giving your answer in its simplest form.

[3]

Oct/Nov 2017 (12)

6.

Functions  $f$  and  $g$  are defined, for  $x > 0$ , by

$$f(x) = \ln x,$$

$$g(x) = 2x^2 + 3.$$

- (i) Write down the range of  $f$ . [1]
- (ii) Write down the range of  $g$ . [1]
- (iii) Find the exact value of  $f^{-1}g(4)$ . [2]
- (iv) Find  $g^{-1}(x)$  and state its domain. [3]