Trigonometry

(Past Year Topical Questions 2012-2017)

May/June 2012 (11)

6.

(i) Given that $15\cos^2\theta + 2\sin^2\theta = 7$, show that $\tan^2\theta = \frac{8}{5}$. [4]

(ii) Solve $15\cos^2\theta + 2\sin^2\theta = 7$ for $0 \leqslant \theta \leqslant \pi$ radians. [3]

May/June 2012 (12)

3.

Show that $\cot A + \dfrac{\sin A}{1 + \cos A} = \operatorname{cosec} A$. [4]

Oct/Nov 2012 (11)

11.

(a) Solve $\operatorname{cosec}\left(2x - \frac{\pi}{3}\right) = \sqrt{2}$ for $0 < x < \pi$ radians. [4]

(b) **(i)** Given that $5(\cos y + \sin y)(2\cos y - \sin y) = 7$, show that $12\tan^2 y - 5\tan y - 3 = 0$. [4]

(ii) Hence solve $5(\cos y + \sin y)(2\cos y - \sin y) = 7$ for $0° < x < 180°$. [3]

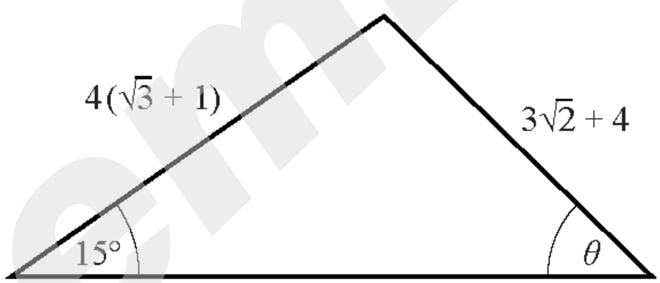
Oct/Nov 2012 (12)

3.

(i) Show that $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta.$ [5]

(ii) Explain why the equation $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1}{2}$ has no solution. [1]

6.



Using $\sin 15^\circ = \dfrac{\sqrt{2}}{4}(\sqrt{3}-1)$ and without using a calculator, find the value of $\sin\theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

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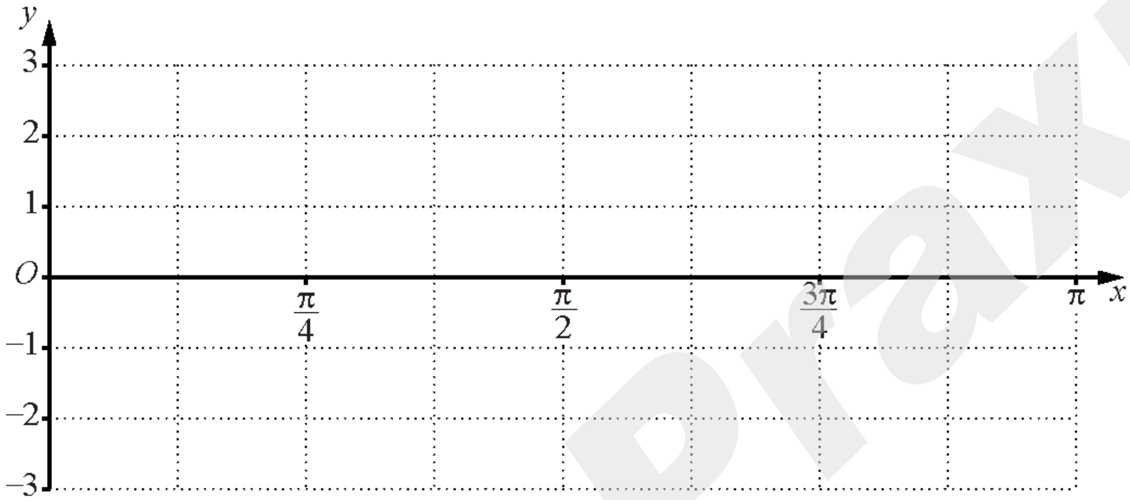
9.

(a) **(i)** Using the axes below, sketch for $0 \leqslant x \leqslant \pi$, the graphs of

$$y = \sin 2x \quad \text{and} \quad y = 1 + \cos 2x.$$ [4]



(ii) Write down the solutions of the equation $\sin 2x - \cos 2x = 1$, for $0 \leqslant x \leqslant \pi$. [2]

(b) **(i)** Write down the amplitude and period of $5\cos 4x - 3$. [2]

(ii) Write down the period of $4\tan 3x$. [1]

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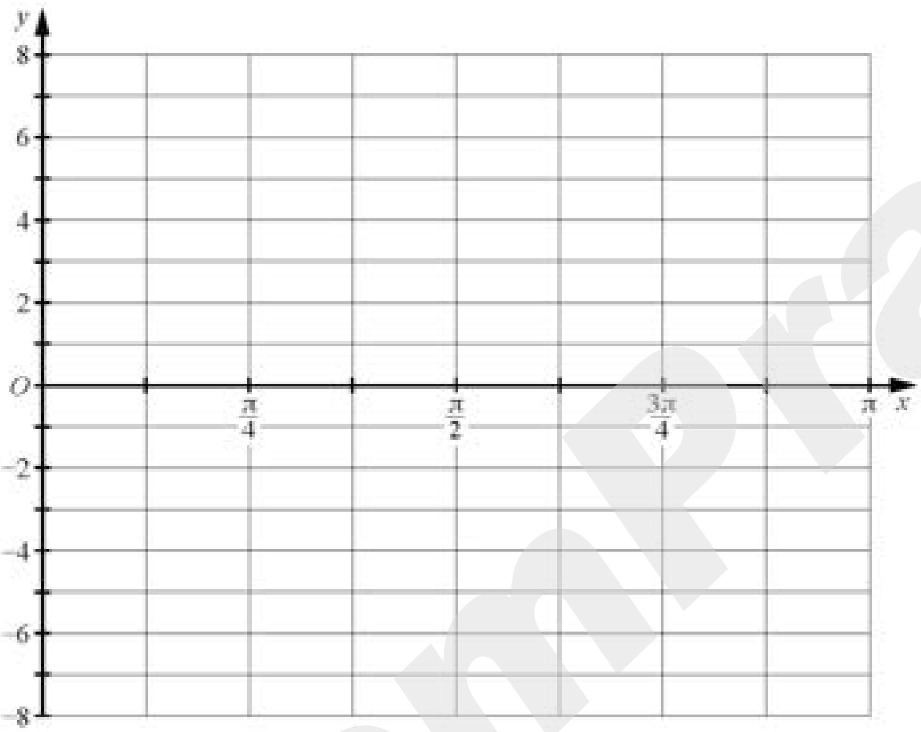
4.

(i) On the axes below sketch, for $0 \leq x \leq \pi$, the graphs of

$$y = \tan x \quad \text{and} \quad y = 1 + 3\sin 2x. \qquad [3]$$



Write down

(ii) the coordinates of the stationary points on the curve $y = 1 + 3\sin 2x$ for $0 \leq x \leq \pi$, [2]

(iii) the number of solutions of the equation $\tan x = 1 + 3\sin 2x$ for $0 \leq x \leq \pi$. [1]

10.

(i) Solve $\tan^2 x - 2\sec x + 1 = 0$ for $0° \leqslant x \leqslant 360°$. **[4]**

(ii) Solve $\cos^2 3y = 5\sin^2 3y$ for $0 \leqslant y \leqslant 2$ radians. **[4]**

(iii) Solve $2\operatorname{cosec}\left(z + \frac{\pi}{4}\right) = 5$ for $0 \leqslant z \leqslant 6$ radians. [4]

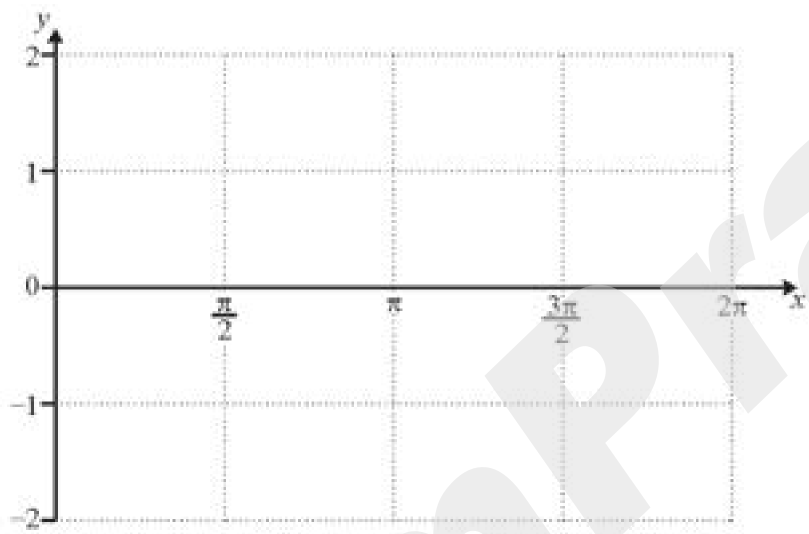
May/June 2013 (11)

1.

On the axes below sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of

(i) $y = \cos x - 1$, [2]

(ii) $y = \sin 2x$. [2]



(iii) State the number of solutions of the equation $\cos x - \sin 2x = 1$, for $0 \leqslant x \leqslant 2\pi$. [1]

11.

(a) Solve $2\sin\left(x+\frac{\pi}{3}\right)=-1$ for $0 \leqslant x \leqslant 2\pi$ radians. [4]

(b) Solve $\tan y - 2 = \cot y$ for $0^\circ \leqslant y \leqslant 180^\circ$. [6]

May/June 2013 (12)

3.

Show that $(1-\cos\theta-\sin\theta)^2 - 2(1-\sin\theta)(1-\cos\theta) = 0$. [3]

11.

(a) Solve $\cos 2x + 2\sec 2x + 3 = 0$ for $0^\circ \leqslant x \leqslant 360^\circ$. [5]

(b) Solve $2\sin^2\left(y - \frac{\pi}{6}\right) = 1$ for $0 \leqslant y \leqslant \pi$. [4]

May/June 2013 (13)

11.

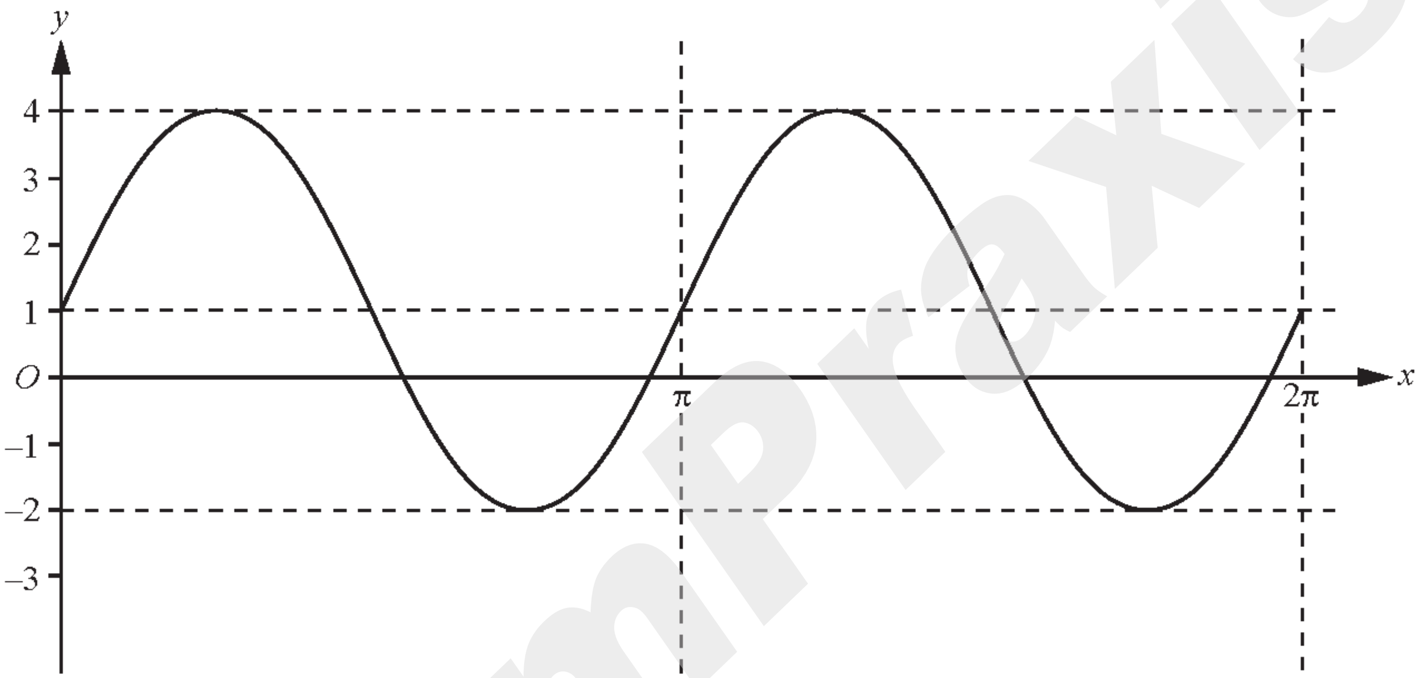
(a) Solve $2\sin\left(x+\frac{\pi}{3}\right)=-1$ for $0 \leqslant x \leqslant 2\pi$ radians. [4]

(b) Solve $\tan y - 2 = \cot y$ for $0^\circ \leqslant y \leqslant 180^\circ$. [6]

Oct/Nov 2013 (11)

1.

The diagram shows the graph of $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive integers.



State the value of a, of b and of c. [3]

$a =$ $b =$ $c =$

3.

Show that $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = 2\sec\theta$. [4]

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Oct/Nov 2013 (13)

3.

Show that $\tan^2\theta - \sin^2\theta = \sin^4\theta\sec^2\theta$. [4]

9.

(a) **(i)** Solve $6\sin^2 x = 5 + \cos x$ for $0° < x < 180°$. [4]

(ii) Hence, or otherwise, solve $6\cos^2 y = 5 + \sin y$ for $0° < y < 180°$. [3]

(b) Solve $4\cot^2 z - 3\cot z = 0$ for $0 < z < \pi$ radians. [4]

May/June 2014 (11)

1.

Show that $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta$. [4]

11.

(a) Solve $5\sin 2x + 3\cos 2x = 0$ for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

(b) Solve $2\cot^2 y + 3\operatorname{cosec} y = 0$ for $0^\circ \leqslant y \leqslant 360^\circ$. [4]

(c) Solve $3\cos(z+1.2) = 2$ for $0 \leqslant z \leqslant 6$ radians. [4]

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May/June 2014 (12)

1.

Show that $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}$ can be written in the form $p\sec A$, where p is an integer to be found. [4]

11.

(a) Solve $\tan^2 x + 5\tan x = 0$ for $0° \leqslant x \leqslant 180°$. [3]

(b) Solve $2\cos^2 y - \sin y - 1 = 0$ for $0° \leqslant y \leqslant 360°$. [4]

(c) Solve $\sec\left(2z - \dfrac{\pi}{6}\right) = 2$ for $0 \leqslant z \leqslant \pi$ radians. [4]

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9.

Solve

(i) $3\sin x\cos x = 2\cos x$ for $0° \leqslant x \leqslant 180°$, [4]

(ii) $10\sin^2 y + \cos y = 8$ for $0° \leqslant y \leqslant 360°$. [5]

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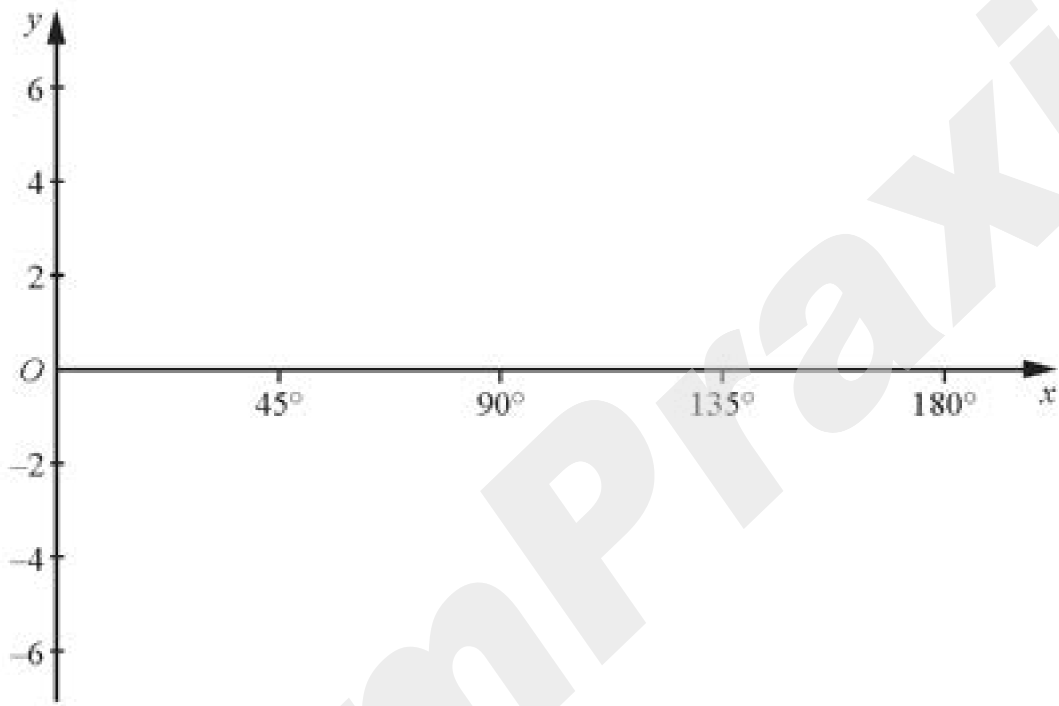
2

(a) On the axes below, sketch the curve $y = 3\cos 2x - 1$ for $0° \leqslant x \leqslant 180°$. [3]



(b) **(i)** State the amplitude of $1 - 4\sin 2x$. [1]

(ii) State the period of $5\tan 3x + 1$. [1]

11.

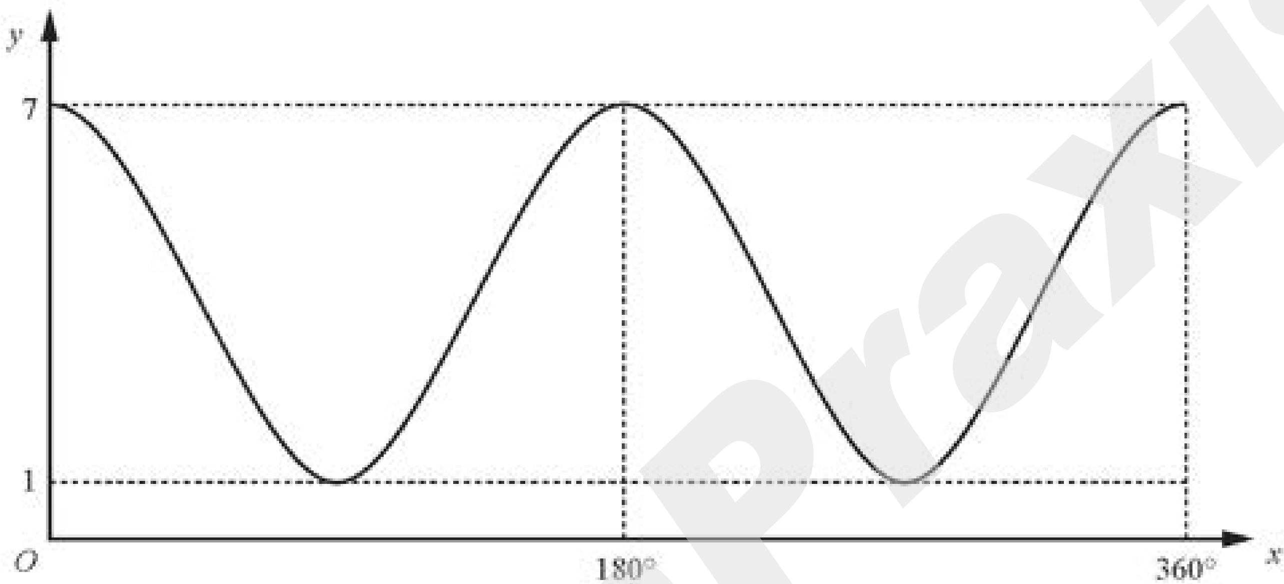
(a) Solve $2\cos 3x = \cot 3x$ for $0° \leqslant x \leqslant 90°$. [5]

(b) Solve $\sec\left(y + \frac{\pi}{2}\right) = -2$ for $0 \leqslant y \leqslant \pi$ radians. [4]

Oct/Nov 2014 (13)

1.

The diagram shows the graph of $y = a\cos bx + c$ for $0° \leqslant x \leqslant 360°$, where a, b and c are positive integers.



State the value of each of a, b and c. [3]

$a =$

$b =$

$c =$

4.

(a) Solve $3\sin x + 5\cos x = 0$ for $0° \leqslant x \leqslant 360°$. [3]

(b) Solve $\operatorname{cosec}\left(3y + \frac{\pi}{4}\right) = 2$ for $0 \leqslant y \leqslant \pi$ radians. [5]

May/June 2015 (11)

1.

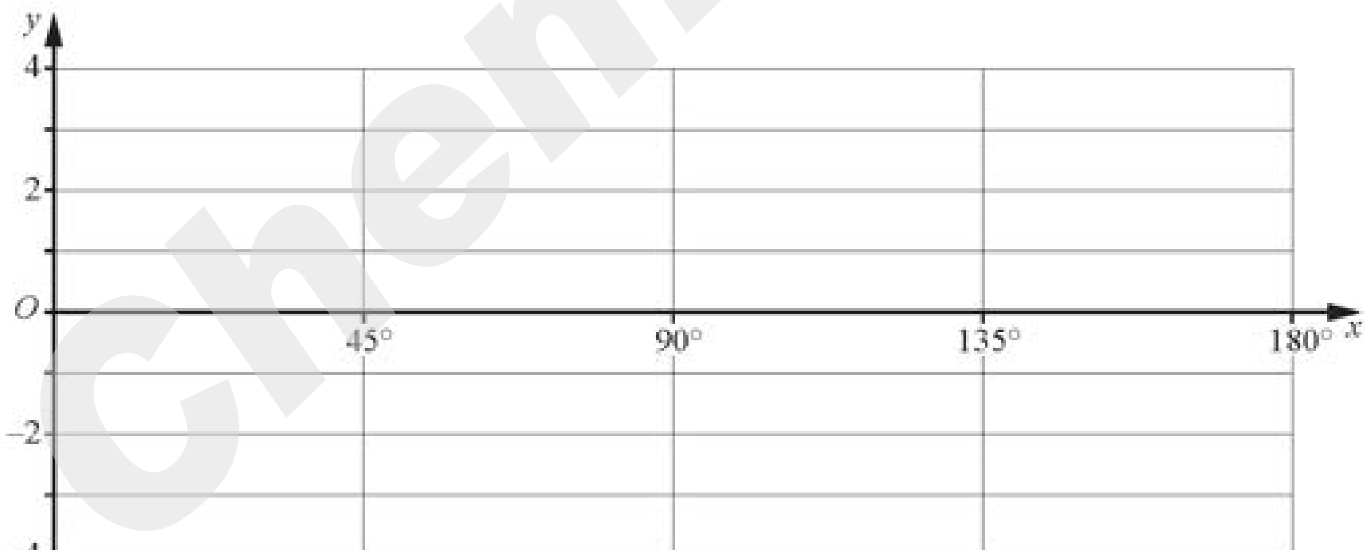
(i) State the period of $\sin 2x$. [1]

(ii) State the amplitude of $1 + 2\cos 3x$. [1]

(iii) On the axes below, sketch the graph of

(a) $y = \sin 2x$ for $0° \leqslant x \leqslant 180°$, [1]

(b) $y = 1 + 2\cos 3x$ for $0° \leqslant x \leqslant 180°$. [2]



(iv) State the number of solutions of $\sin 2x - 2\cos 3x = 1$ for $0° \leqslant x \leqslant 180°$. [1]

10.

(a) Solve $4\sin x = \operatorname{cosec} x$ for $0° \leqslant x \leqslant 360°$. [3]

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$ for $0° \leqslant y \leqslant 180°$. [6]

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$ for $0 \leqslant z \leqslant 2\pi$ radians. [3]

May/June 2015 (12)

2.

Show that $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$. [4]

10.

(a) Solve $2\cos 3x = \sec 3x$ for $0° \leqslant x \leqslant 120°$. [3]

(b) Solve $3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$. [5]

(c) Solve $2\sin\left(z + \dfrac{\pi}{3}\right) = 1$ for $0 \leqslant z \leqslant 2\pi$ radians. [4]

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10.

(a) Solve $4\sin x = \operatorname{cosec} x$ for $0° \leqslant x \leqslant 360°$. [3]

(b) Solve $\tan^2 3y - 2\sec 3y - 2 = 0$ for $0° \leqslant y \leqslant 180°$. [6]

(c) Solve $\tan\left(z - \frac{\pi}{3}\right) = \sqrt{3}$ for $0 \leqslant z \leqslant 2\pi$ radians. [3]

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3.

Show that $\sqrt{\sec^2\theta - 1} + \sqrt{\operatorname{cosec}^2\theta - 1} = \sec\theta \operatorname{cosec}\theta$. [5]

Oct/Nov 2015 (13)

2.

Solve $2\cos^2\left(3x - \frac{\pi}{4}\right) = 1$ for $0 \leqslant x \leqslant \frac{\pi}{3}$. [4]

May/June 2016 (11)

9.

(i) Show that $2\cos x \cot x + 1 = \cot x + 2\cos x$ can be written in the form $(a\cos x - b)(\cos x - \sin x) = 0$, where a and b are constants to be found. [4]

(ii) Hence, or otherwise, solve $2\cos x \cot x + 1 = \cot x + 2\cos x$ for $0 < x < \pi$. [3]

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May/June 2016 (12)

5.

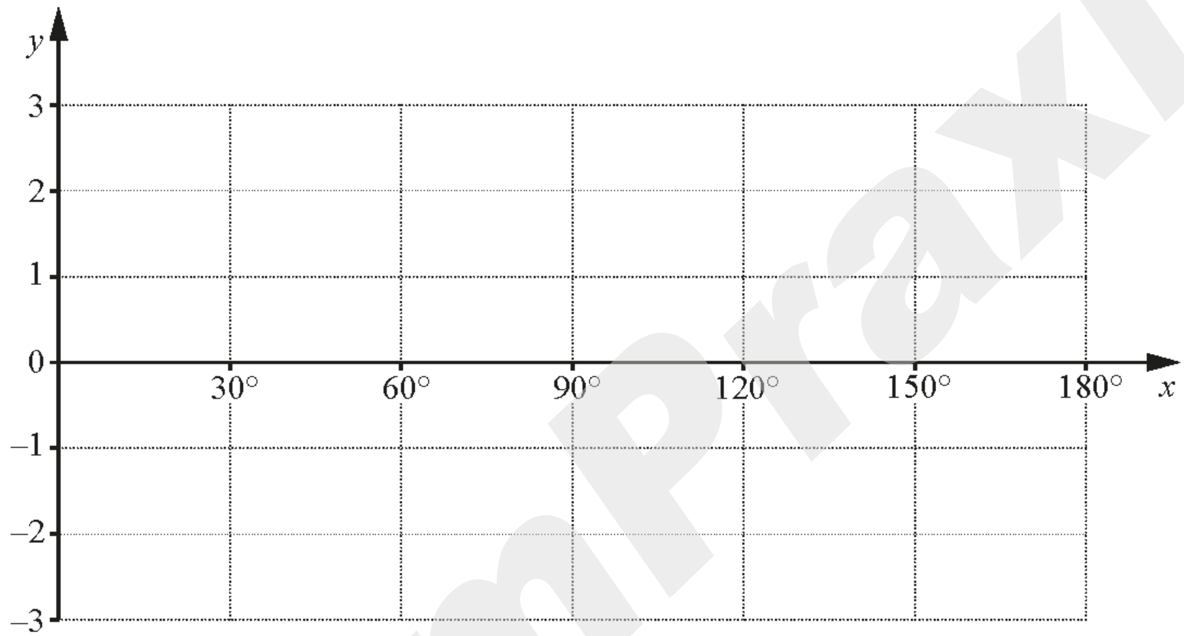
(i) Show that $(1-\cos\theta)(1+\sec\theta)=\sin\theta\tan\theta$. [4]

(ii) Hence solve the equation $(1-\cos\theta)(1+\sec\theta)=\sin\theta$ for $0 \leqslant \theta \leqslant \pi$ radians. [3]

Oct/Nov 2016 (13)

1.

On the axes below, sketch the graph of $y = |2\cos 3x|$ for $0 \leqslant x \leqslant 180°$. [3]



8.

(a) **(i)** Show that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$. [3]

(ii) Hence solve $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = 4$ for $0° < \theta < 360°$. [3]

(b) Solve $\sqrt{3}\tan\left(x + \dfrac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$, giving your answers in terms of π. [3]

May/June 2017 (12)

6.

(i) Show that $\dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$. [4]

It is given that $\displaystyle\int_0^a \frac{\operatorname{cosec} 2\theta}{\cot 2\theta + \tan 2\theta}\,d\theta = \frac{\sqrt{3}}{4}$, where $0 < a < \frac{\pi}{4}$.

(ii) Using your answer to part **(i)** find the value of a, giving your answer in terms of π. [4]

May/June 2017 (13)

2.

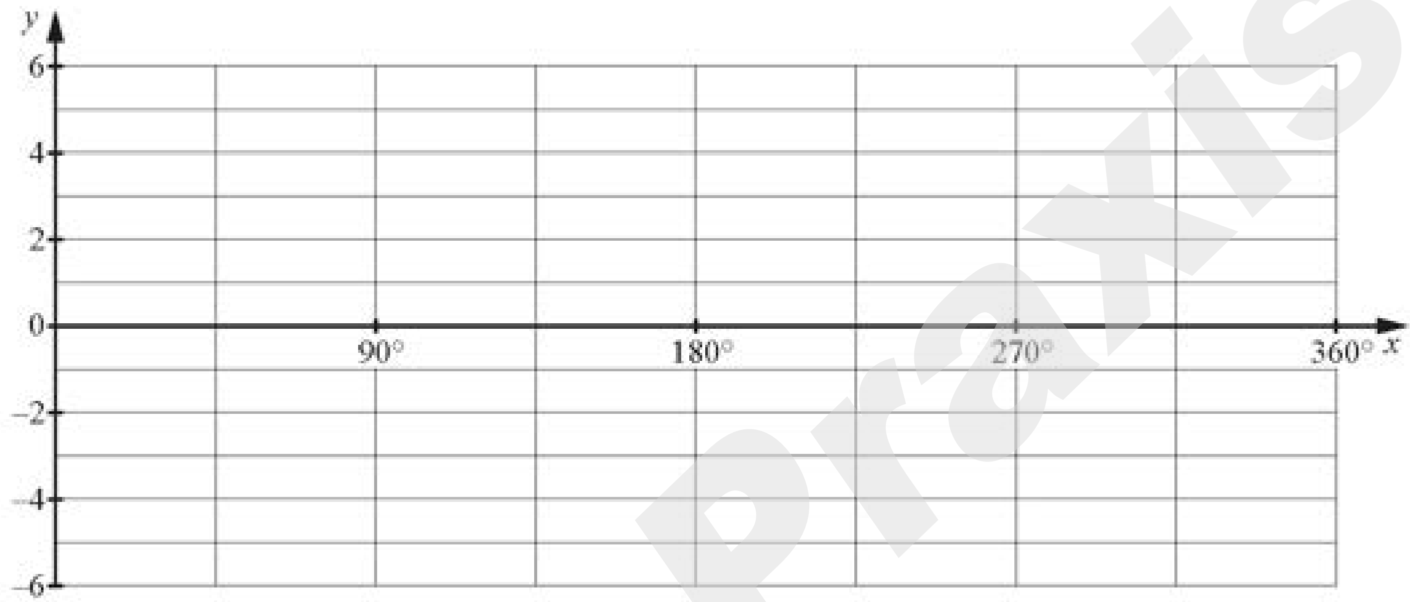
Given that $y = 3 + 4\cos 9x$, write down

(i) the amplitude of y, [1]

(ii) the period of y. [1]

3.

(i) On the axes below, sketch the graph of $y = 3\sin x - 2$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]



(ii) Given that $0 \leqslant |3\sin x - 2| \leqslant k$ for $0^\circ \leqslant x \leqslant 360^\circ$, write down the value of k. [1]

7.

(a) Show that $\dfrac{\tan^2\theta + \sin^2\theta}{\cos\theta + \sec\theta} = \tan\theta\sin\theta$. [4]

(b) Given that $x = 3\sin\phi$ and $y = \dfrac{3}{\cos\phi}$, find the numerical value of $9y^2 - x^2y^2$. [3]

Oct/Nov 2017 (11)

10.

(a) Solve $3\operatorname{cosec} 2x - 4\sin 2x = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Solve $3\tan\left(y - \frac{\pi}{4}\right) = \sqrt{3}$ for $0 \leqslant y \leqslant 2\pi$ radians, giving your answers in terms of π. [4]

Oct/Nov 2017 (12)

2.

The graph of $y = a\sin(bx) + c$ has an amplitude of 4, a period of $\frac{\pi}{3}$ and passes through the point $\left(\frac{\pi}{12}, 2\right)$. Find the value of each of the constants a, b and c. [4]

11.

(a) Solve $2\cot(\phi + 35^\circ) = 5$ for $0^\circ \leqslant \phi \leqslant 360^\circ$. [4]

(b) **(i)** Show that $\frac{\sec\theta}{\cot\theta + \tan\theta} = \sin\theta$. [3]

(ii) Hence solve $\frac{\sec 3\theta}{\cot 3\theta + \tan 3\theta} = -\frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$, giving your answers in terms of π. [4]

Oct/Nov 2017 (13)

1.

Given that $y = 2\sec^2\theta$ and $x = \tan\theta - 5$, express y in terms of x. [2]

4.

The graph of $y = a\cos(bx) + c$ has an amplitude of 3, a period of $\frac{\pi}{4}$ and passes through the point $\left(\frac{\pi}{12}, \frac{5}{2}\right)$. Find the value of each of the constants a, b and c. [4]