Differentiation and integration

(Past Year Topical Questions 2012-2017)

May/June 2012 (11)

5.

(i) Find the equation of the tangent to the curve $y = x^3 + 2x^2 - 3x + 4$ at the point where the curve crosses the y-axis. [4]

(ii) Find the coordinates of the point where this tangent meets the curve again. [3]

10.
Variables x and y are such that $y = e^{2x} + e^{-2x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) By using the substitution $u = e^{2x}$, find the value of y when $\frac{dy}{dx} = 3$. [4]

(iii) Given that x is decreasing at the rate of 0.5 units s^{-1}, find the corresponding rate of change of y when $x = 1$. [3]

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1.

(i) Find $\int \sqrt{7x-5}\,dx$. [3]

(ii) Hence evaluate $\int_2^3 \sqrt{7x-5}\,dx$. [2]

5.
Differentiate the following with respect to x.

(i) $(2 - x^2)\ln(3x + 1)$ [3]

(ii) $\dfrac{4 - \tan 2x}{5x}$ [3]

10.

(a) It is given that $f(x) = \dfrac{1}{2+x}$ for $x \neq -2, x \in \mathbb{R}$.

(i) Find $f''(x)$. [2]

(ii) Find $f^{-1}(x)$. [2]

(iii) Solve $f^2(x) = -1$. [3]

(b) The functions g, h and k are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{1}{x+5}, \quad x \neq -5,$$

$$h(x) = x^2 - 1,$$

$$k(x) = 2x + 1.$$

Express the following in terms of g, h and/or k.

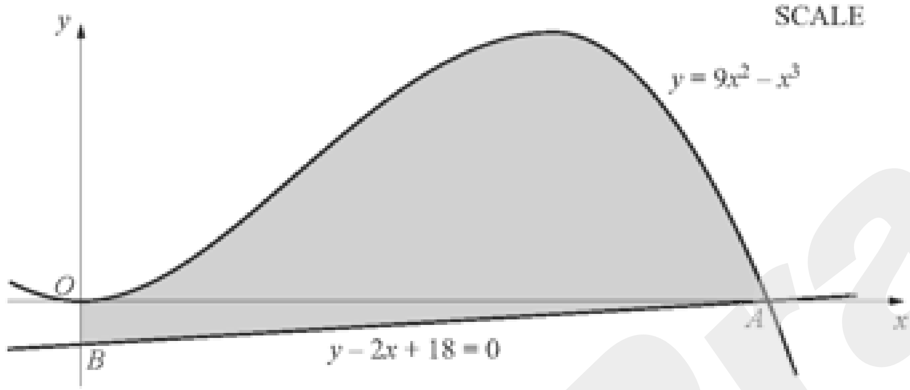
(i) $\dfrac{1}{(x^2-1)+5}$ [1]

(ii) $\dfrac{2}{x+5} + 1$ [1]

11.

EITHER

The diagram shows part of the curve $y = 9x^2 - x^3$, which meets the x-axis at the origin O and at the point A. The line $y - 2x + 18 = 0$ passes through A and meets the y-axis at the point B.

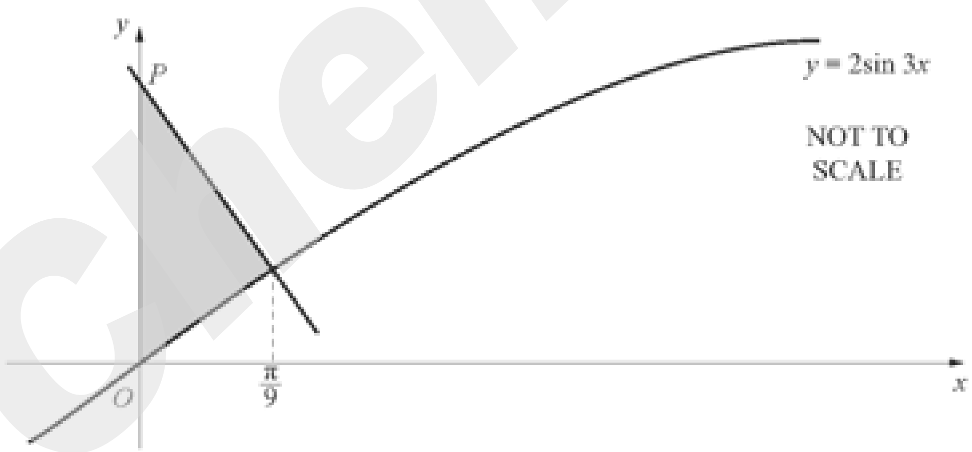


(i) Show that, for $x \geqslant 0$, $9x^2 - x^3 \leqslant 108$. [4]

(ii) Find the area of the shaded region bounded by the curve, the line AB and the y-axis. [6]

OR

The diagram shows part of the curve $y = 2\sin 3x$. The normal to the curve $y = 2\sin 3x$ at the point where $x = \frac{\pi}{9}$ meets the y-axis at the point P.



(i) Find the coordinates of P. [5]

(ii) Find the area of the shaded region bounded by the curve, the normal and the y-axis. [5]

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5.

(i) Find the equation of the tangent to the curve $y = x^3 + 2x^2 - 3x + 4$ at the point where the curve crosses the y-axis. [4]

(ii) Find the coordinates of the point where this tangent meets the curve again. [3]

10.

Variables x and y are such that $y = e^{2x} + e^{-2x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) By using the substitution $u = e^{2x}$, find the value of y when $\frac{dy}{dx} = 3$. [4]

(iii) Given that x is decreasing at the rate of 0.5 units s^{-1}, find the corresponding rate of change of y when $x = 1$. [3]

Oct/Nov 2012 (11)

5.

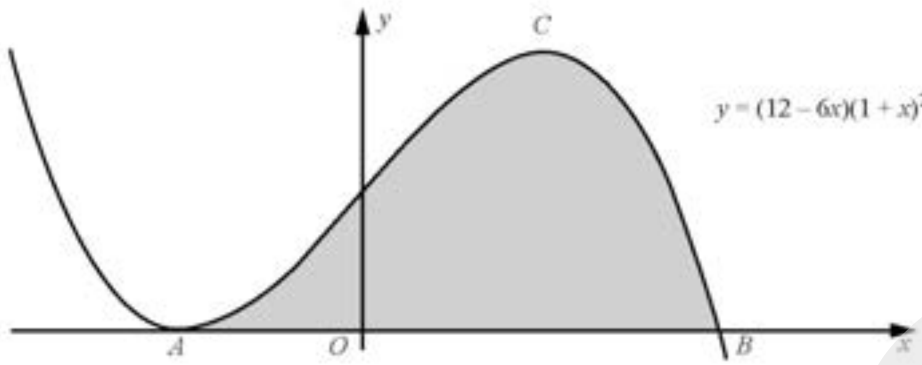
Given that $y = \dfrac{x^2}{\cos 4x}$, find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the approximate change in y when x increases from $\dfrac{\pi}{4}$ to $\dfrac{\pi}{4} + p$, where p is small. [2]

12.

EITHER

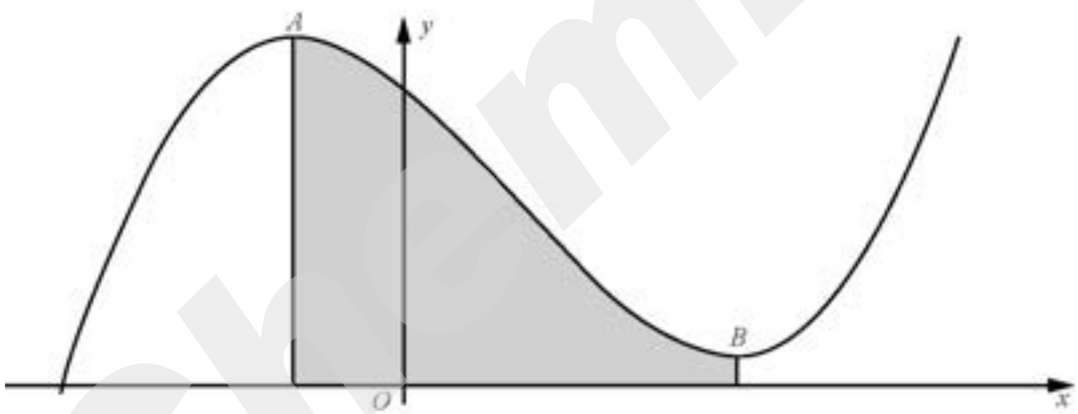


The diagram shows part of the graph of $y = (12 - 6x)(1 + x)^2$, which meets the x-axis at the points A and B. The point C is the maximum point of the curve.

(i) Find the coordinates of each of A, B and C. [6]

(ii) Find the area of the shaded region. [5]

OR



The diagram shows part of a curve such that $\frac{dy}{dx} = 3x^2 - 6x - 9$. Points A and B are stationary points of the curve and lines from A and B are drawn perpendicular to the x-axis. Given that the curve passes through the point (0, 30), find

(i) the equation of the curve, [4]

(ii) the x-coordinate of A and of B, [3]

(iii) the area of the shaded region. [4]

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11.

EITHER

A curve is such that $y = \dfrac{5x^2}{1 + x^2}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{kx}{(1 + x^2)^2}$, where k is an integer to be found. [4]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of this stationary point. [3]

(iii) By using your result from part **(i)**, find $\displaystyle\int \frac{x}{(1 + x^2)^2}\,dx$ and hence evaluate $\displaystyle\int_{-1}^{2} \frac{x}{(1 + x^2)^2}\,dx$. [4]

OR

A curve is such that $y = \dfrac{Ax^2 + B}{x^2 - 2}$, where A and B are constants.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{2x(2A + B)}{(x^2 - 2)^2}$. [4]

It is given that $y = -3$ and $\dfrac{dy}{dx} = -10$ when $x = 1$.

(ii) Find the value of A and of B. [3]

(iii) Using your values of A and B, find the coordinates of the stationary point on the curve, and determine the nature of this stationary point. [4]

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2\.

The rate of change of a variable x with respect to time t is $4\cos^2 t$.

(i) Find the rate of change of x with respect to t when $t = \frac{\pi}{6}$. [1]

The rate of change of a variable y with respect to time t is $3\sin t$.

(ii) Using your result from part **(i)**, find the rate of change of y with respect to x when $t = \frac{\pi}{6}$. [3]

7.

A particle P moves along the x-axis such that its distance, x m, from the origin O at time t s is given by $x = \frac{t}{t^2+1}$ for $t \geqslant 0$.

(i) Find the greatest distance of P from O. [4]

(ii) Find the acceleration of P at the instant when P is at its greatest distance from O. [3]

11.

EITHER

The tangent to the curve $y = 5e^{x} + 3e^{-x}$ at the point where $x = \ln\frac{3}{5}$, meets the x-axis at the point P.

(i) Find the coordinates of P. [5]

The area of the region enclosed by the curve $y = 5e^{x} + 3e^{-x}$, the y-axis, the positive x-axis and the line $x = a$ is 12 square units.

(ii) Show that $5e^{2a} - 14e^{a} - 3 = 0$. [3]

(iii) Hence find the value of a. [3]

OR

(i) Given that $y = \dfrac{3e^{2x}}{1 + e^{2x}}$, show that $\dfrac{dy}{dx} = \dfrac{Ae^{2x}}{(1 + e^{2x})^2}$, where A is a constant to be found. [4]

(ii) Find the equation of the tangent to the curve $y = \dfrac{3e^{2x}}{1 + e^{2x}}$ at the point where the curve crosses the y-axis. [3]

(iii) Using your result from part **(i)**, find $\displaystyle\int \frac{e^{2x}}{(1 + e^{2x})^2}\,dx$ and hence evaluate $\displaystyle\int_0^{\ln 3} \frac{e^{2x}}{(1 + e^{2x})^2}\,dx$. [4]

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5.

(i) Find $\int\left(1-\frac{6}{x^2}\right)\mathrm{d}x$. [2]

(ii) Hence find the value of the positive constant k for which $\int_{k}^{3k}\left(1-\frac{6}{x^2}\right)\mathrm{d}x = 2$. [4]

10.

The point A, whose x-coordinate is 2, lies on the curve with equation $y = x^3 - 4x^2 + x + 1$.

(i) Find the equation of the tangent to the curve at A. [4]

This tangent meets the curve again at the point B.

(ii) Find the coordinates of B. [4]

(iii) Find the equation of the perpendicular bisector of the line AB. [4]

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6.

The normal to the curve $y + 2 = 3\tan x$, at the point on the curve where $x = \dfrac{3\pi}{4}$, cuts the y-axis at the point P. Find the coordinates of P. [6]

10.

(a) **(i)** Find $\int \sqrt{2x-5}\,\mathrm{d}x$. [2]

(ii) Hence evaluate $\int_3^{15} \sqrt{2x-5}\,\mathrm{d}x$. [2]

(i) Find $\frac{d}{dx}(x^3 \ln x)$. [2]

(ii) Hence find $\int x^2 \ln x \, dx$. [3]

12.
A particle P moves in a straight line such that, t s after leaving a point O, its velocity $v\,\mathrm{m\,s^{-1}}$ is given by $v = 36t - 3t^2$ for $t \geqslant 0$.

(i) Find the value of t when the velocity of P stops increasing. [2]

(ii) Find the value of t when P comes to instantaneous rest. [2]

(iii) Find the distance of P from O when P is at instantaneous rest. [3]

(iv) Find the speed of P when P is again at O. [4]

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5.

(i) Find $\int (9 + \sin 3x)\,dx$. [3]

(ii) Hence show that $\int_{\frac{\pi}{9}}^{\pi} (9 + \sin 3x)\,dx = a\pi + b$, where a and b are constants to be found. [3]

9.

(a) Differentiate $4x^3 \ln(2x + 1)$ with respect to x. [3]

(b) **(i)** Given that $y = \frac{2x}{\sqrt{x+2}}$, show that $\frac{dy}{dx} = \frac{x+4}{(\sqrt{x+2})^3}$. [4]

(ii) Hence find $\int \frac{5x + 20}{(\sqrt{x + 2})^3}\,dx$. [2]

(iii) Hence evaluate $\int_2^7 \frac{5x + 20}{(\sqrt{x + 2})^3}\,dx$. [2]

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4.

A curve has equation $y = \dfrac{e^{2x}}{(x+3)^2}$.

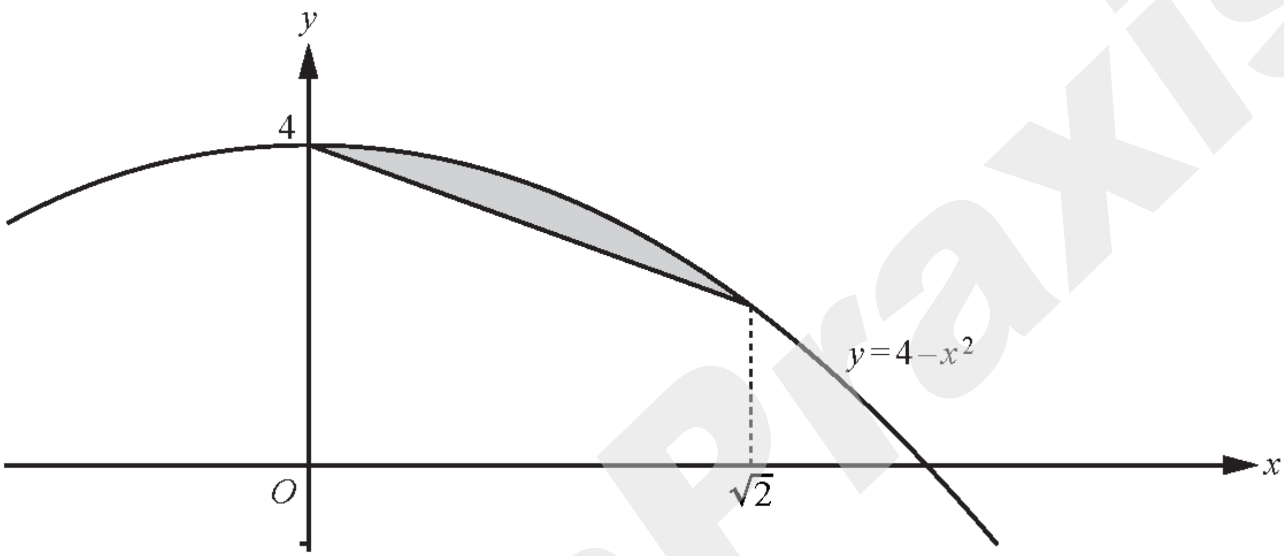
(i) Show that $\dfrac{dy}{dx} = \dfrac{Ae^{2x}(x+2)}{(x+3)^3}$, where A is a constant to be found. [4]

(ii) Find the exact coordinates of the point on the curve where $\dfrac{dy}{dx} = 0$. [2]

6.

Do not use a calculator in this question.

The diagram shows part of the curve $y = 4 - x^2$.



Show that the area of the shaded region can be written in the form $\frac{\sqrt{2}}{p}$, where p is an integer to be found. [6]

11.

(i) Given that $\int_0^k \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) dx = \frac{3}{4}$, where k is a constant, show that

$$4e^{4k} - 12e^{2k} + 5 = 0.$$ [5]

(ii) Using a substitution of $y = e^{2k}$, or otherwise, find the possible values of k. [4]

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5.

(i) Given that $y = e^{x^2}$, find $\frac{dy}{dx}$. [2]

(ii) Use your answer to part **(i)** to find $\int xe^{x^2}\,dx$. [2]

(iii) Hence evaluate $\int_0^2 xe^{x^2}\,dx$. [2]

7.

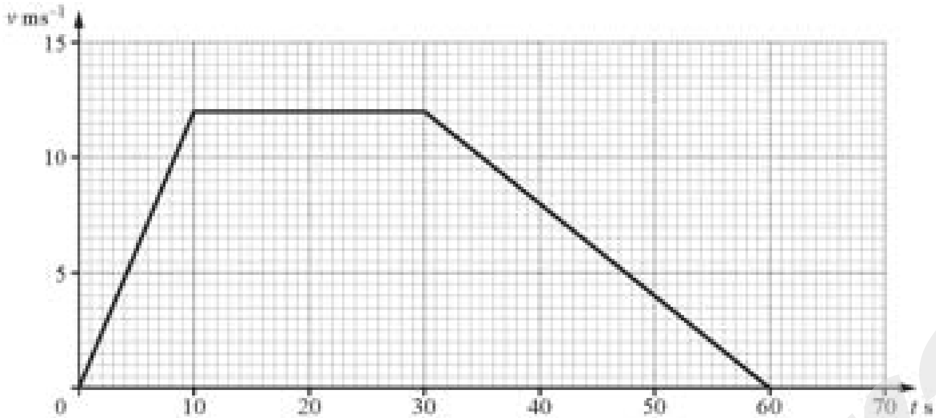
A curve is such that $\frac{dy}{dx} = 4x + \frac{1}{(x+1)^2}$ for $x > 0$. The curve passes through the point $\left(\frac{1}{2}, \frac{5}{6}\right)$.

(i) Find the equation of the curve. [4]

(ii) Find the equation of the normal to the curve at the point where $x = 1$. [4]

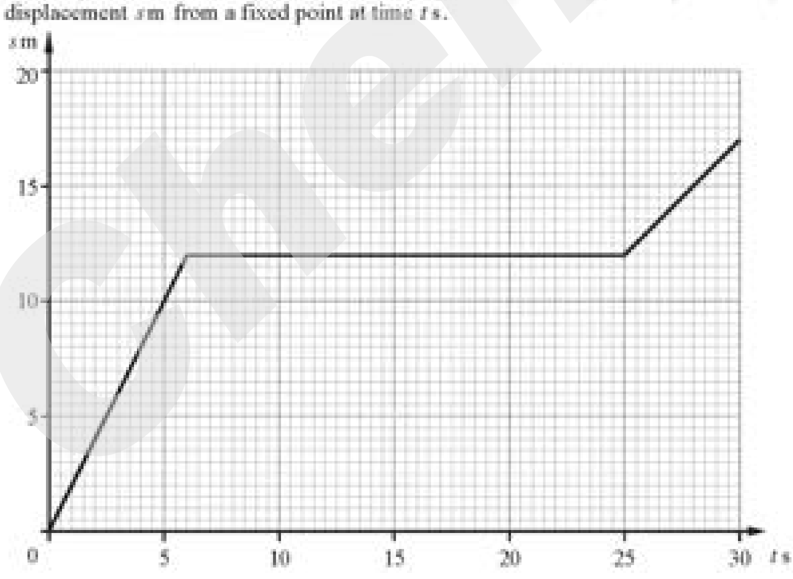
9.

(a) The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity v ms^{-1} at time t s after leaving a fixed point.

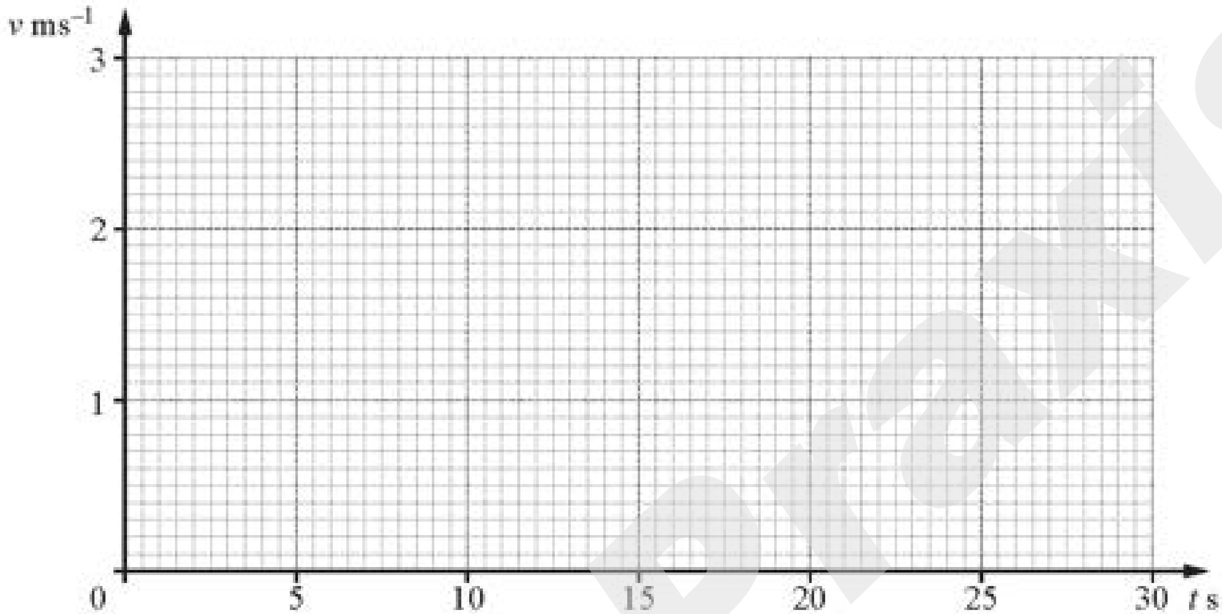


Find the distance travelled by the particle P. [2]

(b) The diagram shows the displacement-time graph of a particle Q moving in a straight line with displacement s m from a fixed point at time t s.



On the axes below, plot the corresponding velocity-time graph for the particle Q. [3]



(c) The displacement s m of a particle R, which is moving in a straight line, from a fixed point at time t s is given by $s = 4t - 16\ln(t+1) + 13$.

(i) Find the value of t for which the particle R is instantaneously at rest. [3]

(ii) Find the value of t for which the acceleration of the particle R is 0.25ms^{-2}. [2]

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4.

The region enclosed by the curve $y = 2\sin 3x$, the x-axis and the line $x = a$, where $0 < a < 1$ radian, lies entirely above the x-axis. Given that the area of this region is $\frac{1}{3}$ square unit, find the value of a. [6]

9.

A solid circular cylinder has a base radius of r cm and a volume of 4000 cm^3.

(i) Show that the total surface area, $A\text{ cm}^2$, of the cylinder is given by $A = \frac{8000}{r} + 2\pi r^2$. [3]

(ii) Given that r can vary, find the minimum total surface area of the cylinder, justifying that this area is a minimum. [6]

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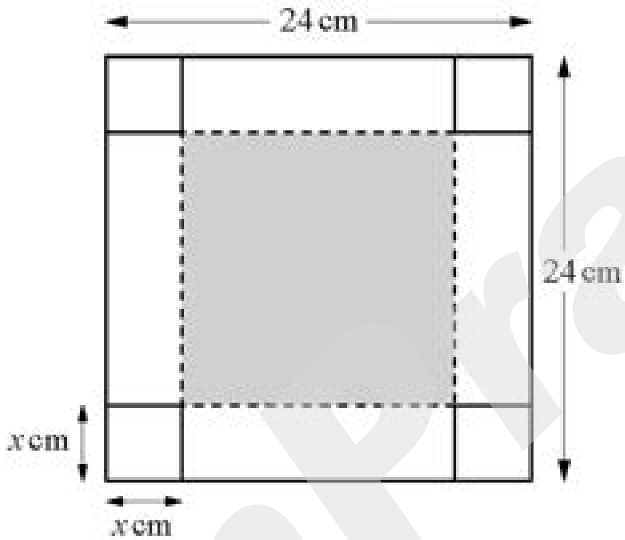
4.

The diagram shows a thin square sheet of metal measuring 24 cm by 24 cm. A square of side x cm is cut off from each corner. The remainder is then folded to form an open box, x cm deep, whose square base is shown shaded in the diagram.



(i) Show that the volume, V cm^3, of the box is given by $V = 4x^3 - 96x^2 + 576x$. [2]

(ii) Given that x can vary, find the maximum volume of the box. [4]

6.

Find the equation of the normal to the curve $y = x(x^2 - 12)^{\frac{1}{3}}$ at the point on the curve where $x = 2$. [6]

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8.

A particle moves in a straight line such that, t s after passing through a fixed point O, its velocity, $v \text{ ms}^{-1}$, is given by $v = 5 - 4e^{-2t}$.

(i) Find the velocity of the particle at O. [1]

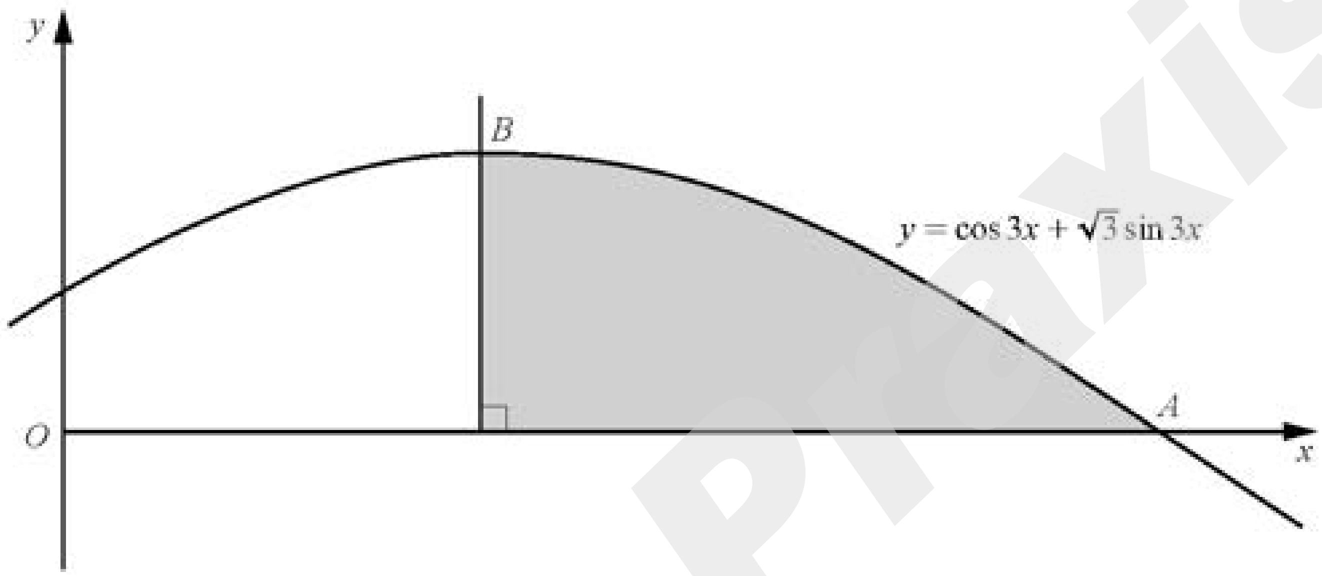
(ii) Find the value of t when the acceleration of the particle is 6 ms^{-2}. [3]

(iii) Find the distance of the particle from O when $t = 1.5$. [5]

(iv) Explain why the particle does not return to O. [1]

11.

The diagram shows the graph of $y = \cos 3x + \sqrt{3}\sin 3x$, which crosses the x-axis at A and has a maximum point at B.



(i) Find the x-coordinate of A. [3]

(ii) Find $\frac{dy}{dx}$ and hence find the x-coordinate of B. [4]

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(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the line through B parallel to the y-axis. [5]

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1.

Find the coordinates of the stationary point on the curve $y = x^2 + \dfrac{16}{x}$. [4]

3.

A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{x+3}}$ for $x > -3$. The curve passes through the point $(6, 10)$.

(i) Find the equation of the curve. [4]

(ii) Find the x-coordinate of the point on the curve where $y = 6$. [1]

5.

(i) Find the equation of the tangent to the curve $y = x^3 - \ln x$ at the point on the curve where $x = 1$. [4]

(ii) Show that this tangent bisects the line joining the points $(-2, 16)$ and $(12, 2)$. [2]

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8.

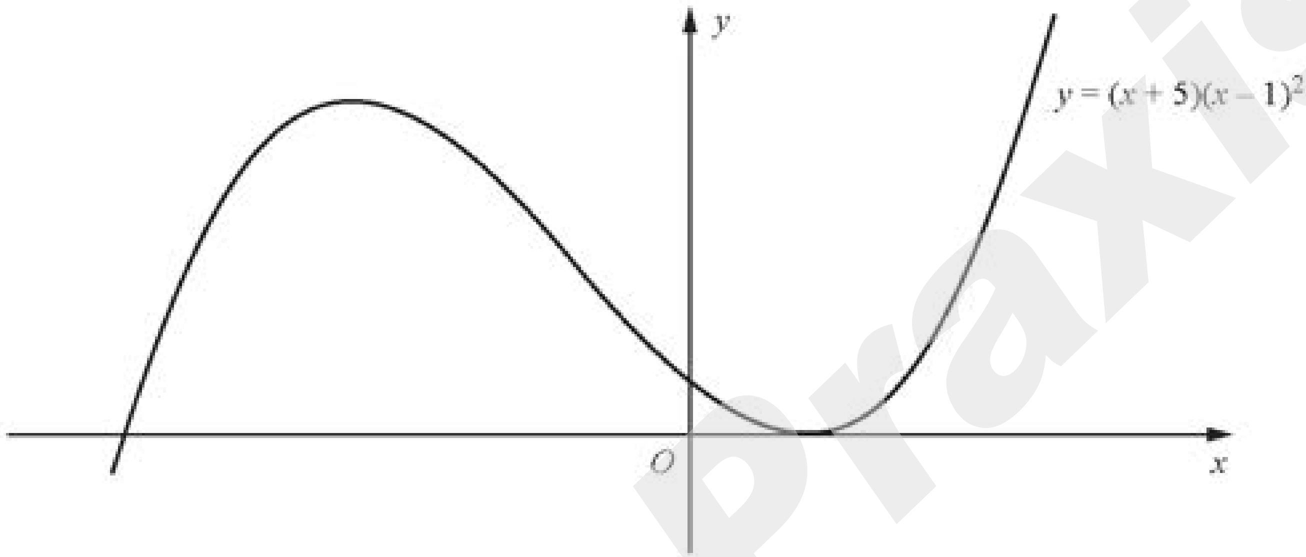
(i) Given that $f(x) = x\ln x^3$, show that $f'(x) = 3(1 + \ln x)$. [3]

(ii) Hence find $\int (1 + \ln x)\,dx$. [2]

(iii) Hence find $\int_1^2 \ln x\,dx$ in the form $p + \ln q$, where p and q are integers. [3]

11.

The diagram shows part of the curve $y = (x + 5)(x - 1)^2$.



(i) Find the x-coordinates of the stationary points of the curve. [5]

(ii) Find $\int (x+5)(x-1)^2 \, dx$. [3]

(iii) Hence find the area enclosed by the curve and the x-axis. [2]

(iv) Find the set of positive values of k for which the equation $(x+5)(x-1)^2 = k$ has only one real solution. [2]

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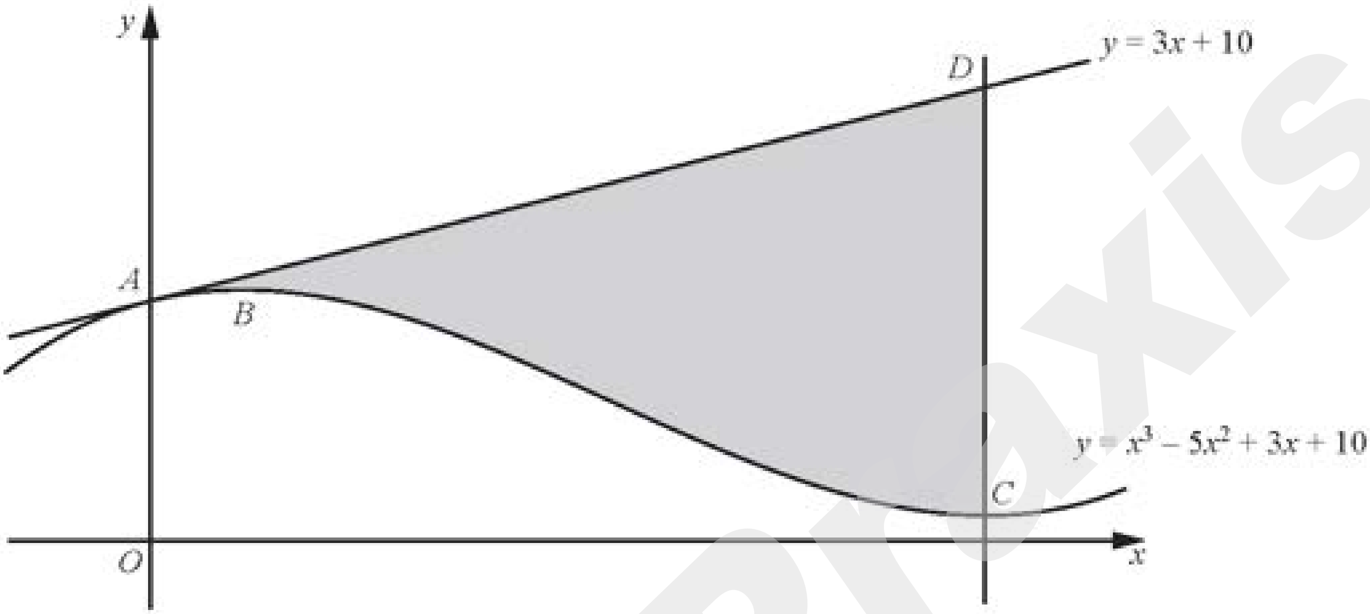
7.

The point A, where $x = 0$, lies on the curve $y = \dfrac{\ln(4x^2+3)}{x-1}$. The normal to the curve at A meets the x-axis at the point B.

(i) Find the equation of this normal. [7]

(ii) Find the area of the triangle AOB, where O is the origin. [2]

9.



The diagram shows parts of the line $y = 3x + 10$ and the curve $y = x^3 - 5x^2 + 3x + 10$. The line and the curve both pass through the point A on the y-axis. The curve has a maximum at the point B and a minimum at the point C. The line through C, parallel to the y-axis, intersects the line $y = 3x + 10$ at the point D.

(i) Show that the line AD is a tangent to the curve at A. [2]

(ii) Find the x-coordinate of B and of C. [3]

(iii) Find the area of the shaded region $ABCD$, showing all your working. [5]

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6.

A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by $x = 10\ln(t^2 + 4) - 4t$.

(i) Find the initial displacement of the particle from O. [1]

(ii) Find the values of t when the particle is instantaneously at rest. [4]

(iii) Find the value of t when the acceleration of the particle is zero. [5]

8.

(i) Find $\int (10e^{2x} + e^{-2x})dx$. [2]

(ii) Hence find $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx$ in terms of the constant k. [2]

(iii) Given that $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx = -60$, show that $11e^{2k} - 11e^{-2k} + 120 = 0$. [2]

(iv) Using a substitution of $y = e^{2k}$ or otherwise, find the value of k in the form $a \ln b$, where a and b are constants. [3]

9.

A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x- and y-axes at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. [8]

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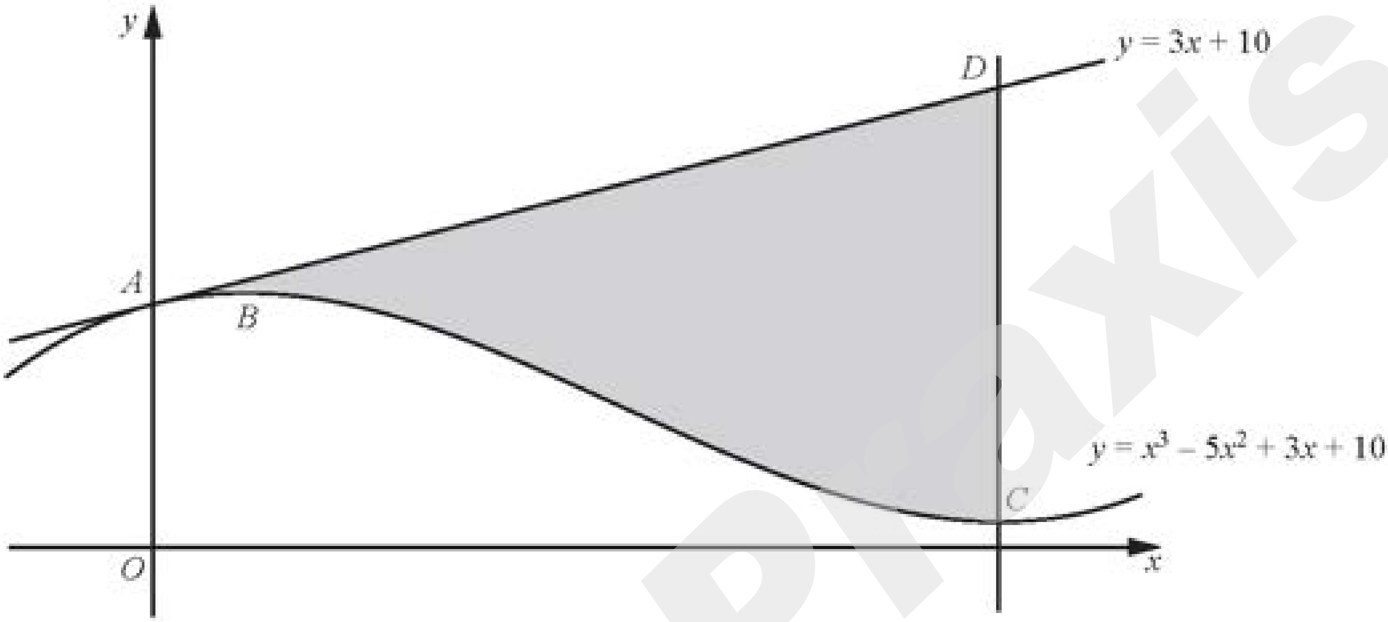
9.



The diagram shows parts of the line $y = 3x + 10$ and the curve $y = x^3 - 5x^2 + 3x + 10$. The line and the curve both pass through the point A on the y-axis. The curve has a maximum at the point B and a minimum at the point C. The line through C, parallel to the y-axis, intersects the line $y = 3x + 10$ at the point D.

(i) Show that the line AD is a tangent to the curve at A. [2]

(ii) Find the x-coordinate of B and of C. [3]

(iii) Find the area of the shaded region $ABCD$, showing all your working. [5]

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2.

A curve, showing the relationship between two variables x and y, passes through the point $P(-1, 3)$.

The curve has a gradient of 2 at P. Given that $\dfrac{d^2y}{dx^2} = -5$, find the equation of the curve. [4]

5.

Variables x and y are such that $y = (x-3)\ln(2x^2+1)$.

(i) Find the value of $\dfrac{dy}{dx}$ when $x = 2$. [4]

(ii) Hence find the approximate change in y when x changes from 2 to 2.03. [2]

8.

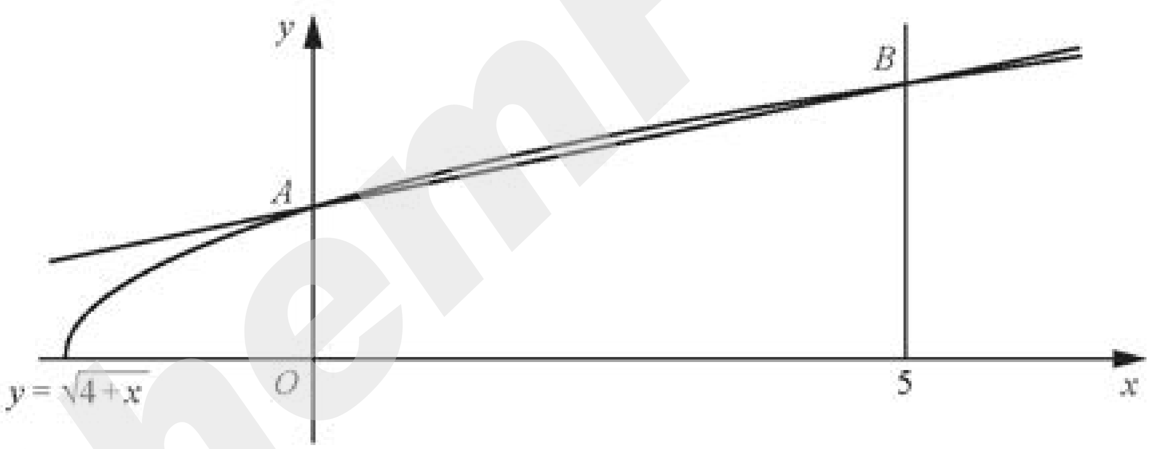
Find the equation of the tangent to the curve $y = \frac{2x-1}{\sqrt{x^2+5}}$ at the point where $x = 2$. [7]

9.

You are not allowed to use a calculator in this question.

(i) Find $\int \sqrt{4+x}\,dx$. [2]

(ii)



The diagram shows the graph of $y = \sqrt{4+x}$, which meets the y-axis at the point A and the line $x = 5$ at the point B. Using your answer to part **(i)**, find the area of the region enclosed by the curve and the straight line AB. [5]

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5.

Find the equation of the normal to the curve $y = 5\tan x - 3$ at the point where $x = \frac{\pi}{4}$. [5]

7.

A curve, showing the relationship between two variables x and y, is such that $\frac{d^2y}{dx^2} = 6\cos 3x$. Given that the curve has a gradient of $4\sqrt{3}$ at the point $\left(\frac{\pi}{9}, -\frac{1}{3}\right)$, find the equation of the curve. [6]

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3.

Find the equation of the normal to the curve $y = \ln(2x^2 - 7)$ at the point where the curve crosses the positive x-axis. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

5.

Do not use a calculator in this question.

(i) Show that $\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = pxe^{4x}$, where p is an integer to be found. [4]

(ii) Hence find the exact value of $\int_0^{\ln 2} xe^{4x}\,dx$, giving your answer in the form $a\ln 2 + \frac{b}{c}$, where a, b and c are integers to be found. [4]

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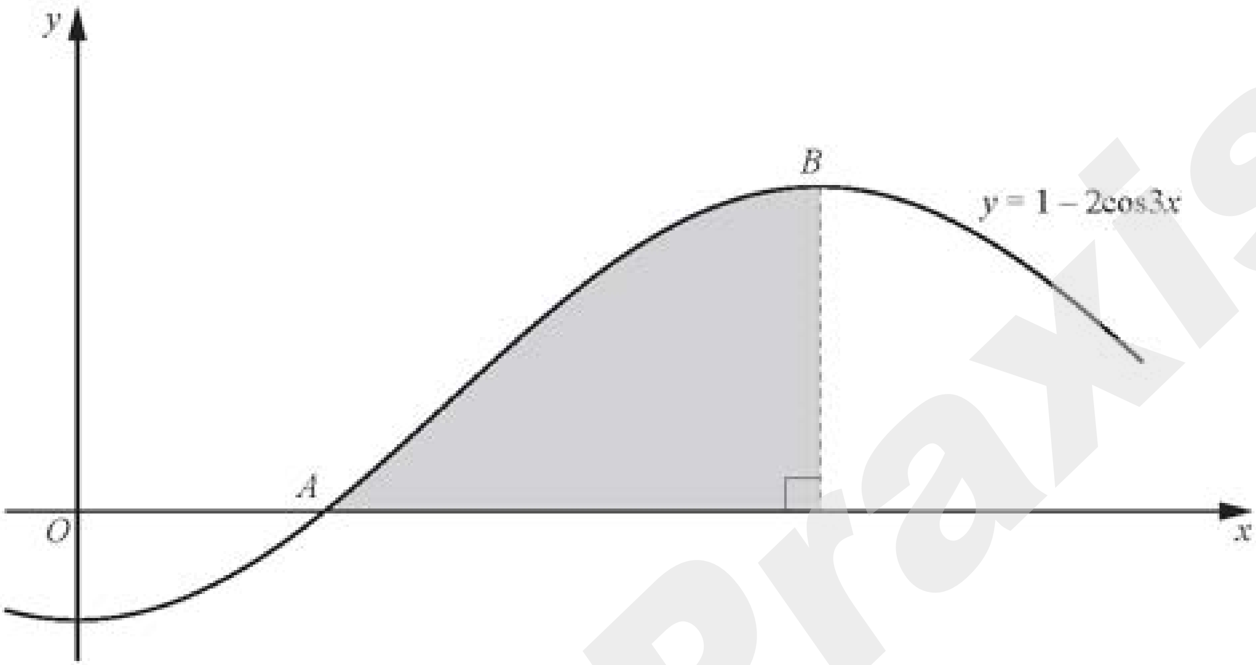
6.

Show that $\frac{d}{dx}(e^{3x}\sqrt{4x+1})$ can be written in the form $\frac{e^{3x}(px+q)}{\sqrt{4x+1}}$, where p and q are integers to be found. [5]

7.



The diagram shows part of the graph of $y = 1 - 2\cos 3x$, which crosses the x-axis at the point A and has a maximum at the point B.

(i) Find the coordinates of A. [2]

(ii) Find the coordinates of B. [2]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the perpendicular from B to the x-axis. [4]

9.

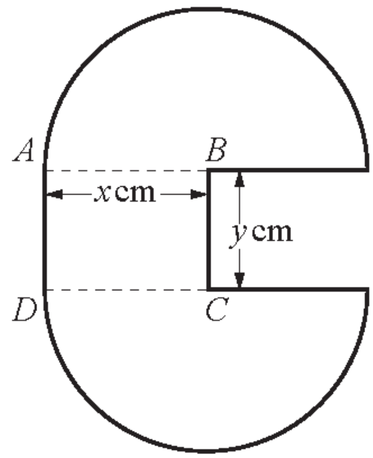
A curve passes through the point $\left(2, -\frac{4}{3}\right)$ and is such that $\frac{dy}{dx} = (3x + 10)^{-\frac{1}{2}}$.

(i) Find the equation of the curve. [4]

The normal to the curve, at the point where $x = 5$, meets the line $y = -\frac{5}{3}$ at the point P.

(ii) Find the x-coordinate of P. [6]

10.



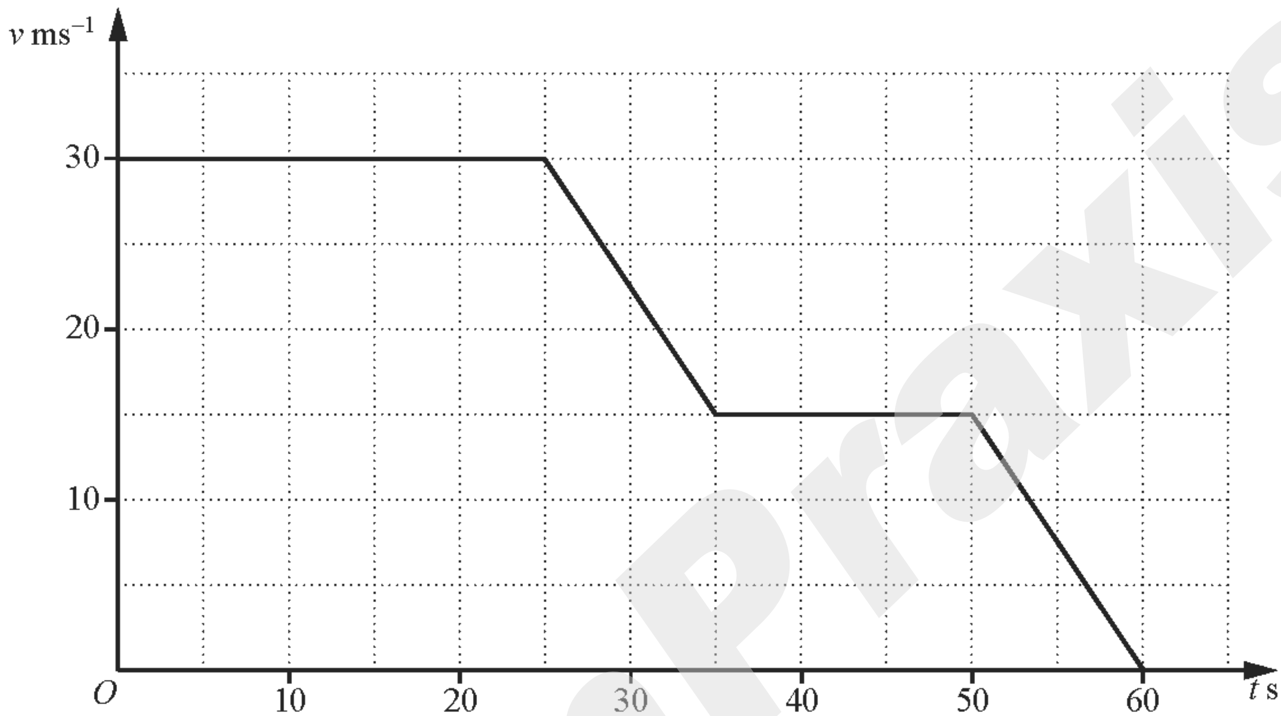
The diagram shows a badge, made of thin sheet metal, consisting of two semi-circular pieces, centres B and C, each of radius x cm. They are attached to each other by a rectangular piece of thin sheet metal, $ABCD$, such that AB and CD are the radii of the semi-circular pieces and $AD = BC = y$ cm.

(i) Given that the area of the badge is $20\,\text{cm}^2$, show that the perimeter, P cm, of the badge is given by $P = 2x + \dfrac{40}{x}$. [4]

(ii) Given that x can vary, find the minimum value of P, justifying that this value is a minimum. [5]

11.

(a)

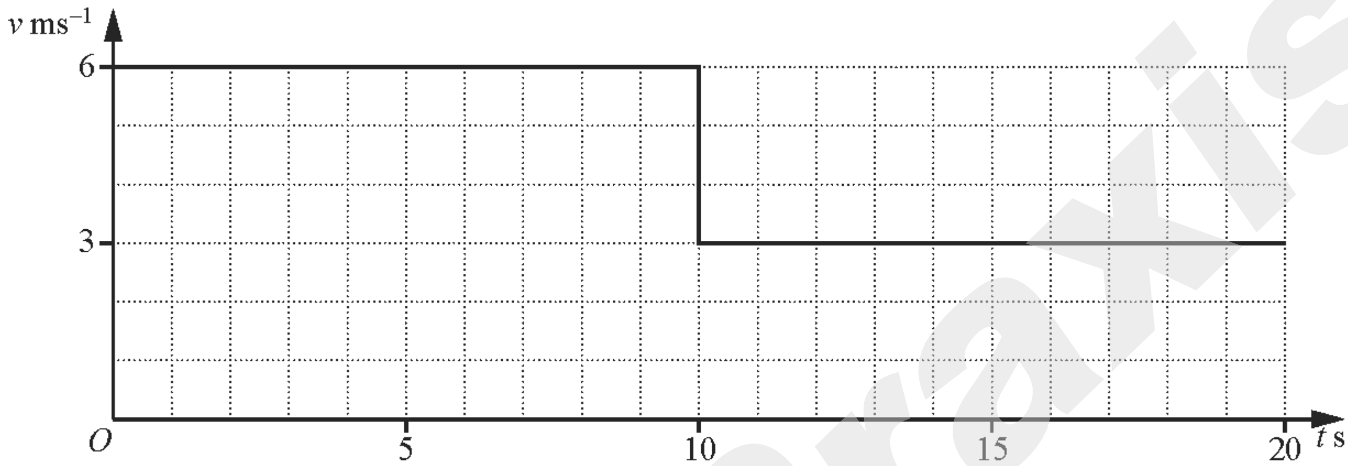


The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity v ms^{-1} at time t s after leaving a fixed point.

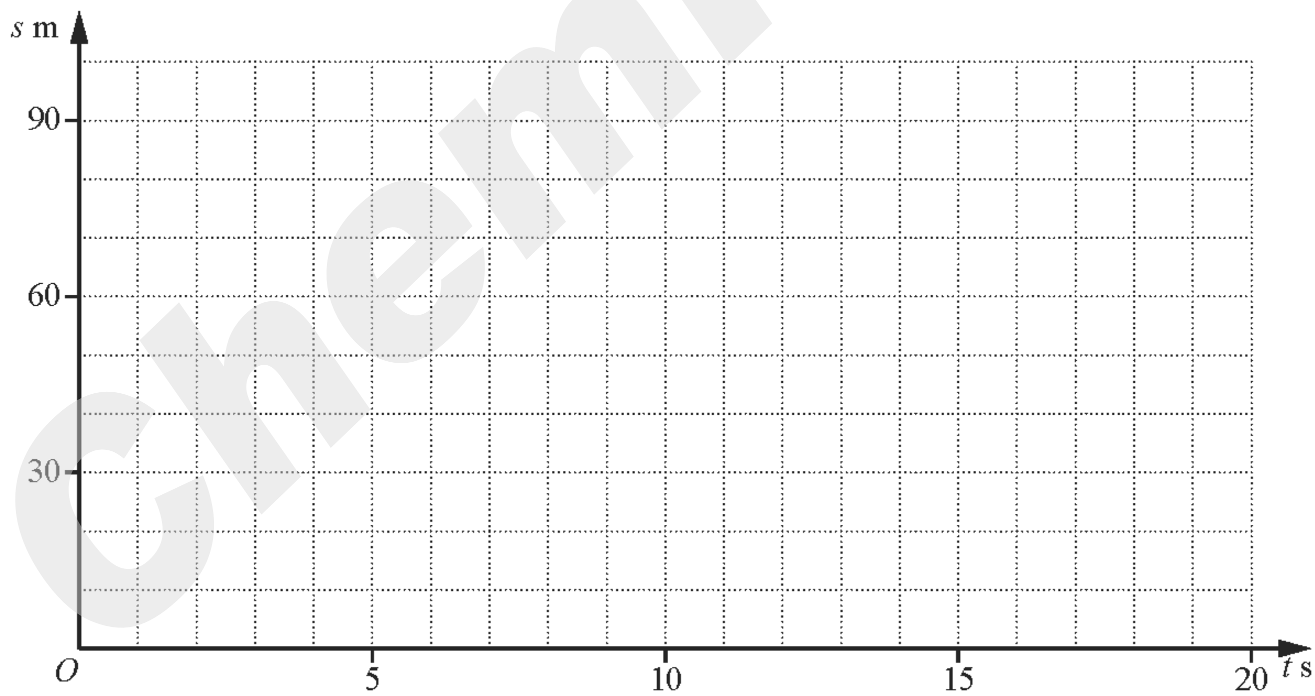
(i) Find the distance travelled by the particle P. [2]

(ii) Write down the deceleration of the particle when $t = 30$. [1]

(b) The diagram shows a velocity-time graph of a particle Q moving in a straight line with velocity v ms^{-1}, at time t s after leaving a fixed point.



The displacement of Q at time t s is s m. On the axes below, draw the corresponding displacement-time graph for Q. [2]



(c) The velocity, v ms^{-1}, of a particle R moving in a straight line, t s after passing through a fixed point O, is given by $v = 4e^{2t} + 6$.

(i) Explain why the particle is never at rest. [1]

(ii) Find the exact value of t for which the acceleration of R is 12 ms^{-2}. [2]

(iii) Showing all your working, find the distance travelled by R in the interval between $t = 0.4$ and $t = 0.5$. [4]

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10.

(i) Given that $f(x) = 4x^3 + kx + p$ is exactly divisible by $x + 2$ and $f'(x)$ is exactly divisible by $2x - 1$, find the value of k and of p. [4]

(ii) Using the values of k and p found in part (i), show that $f(x) = (x+2)(ax^2 + bx + c)$, where a, b and c are integers to be found. [2]

(iii) Hence show that $f(x) = 0$ has only one solution and state this solution. [2]

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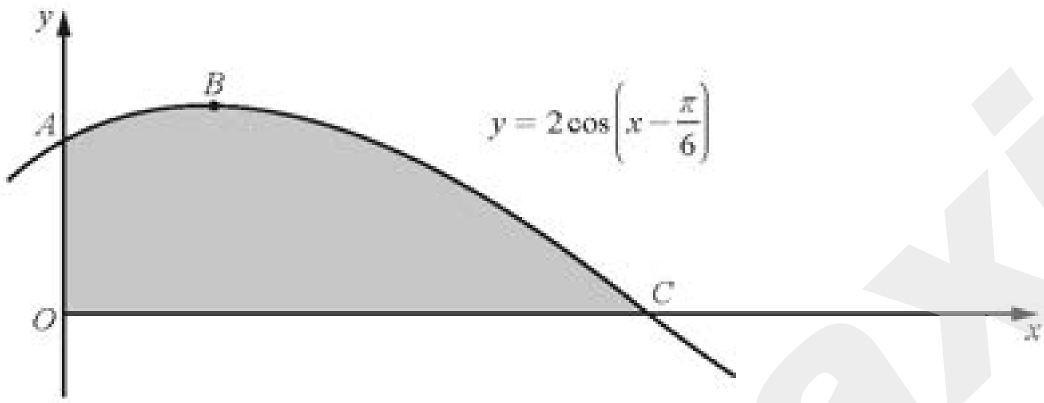
5.

(i) Find the equation of the normal to the curve $y = \frac{1}{2}\ln(3x + 2)$ at the point P where $x = -\frac{1}{3}$. [4]

The normal to the curve at the point P intersects the y-axis at the point Q. The curve $y = \frac{1}{2}\ln(3x + 2)$ intersects the y-axis at the point R.

(ii) Find the area of the triangle PQR. [3]

7.



The diagram shows part of the graph of $y = 2\cos\left(x - \frac{\pi}{6}\right)$. The graph intersects the y-axis at the point A, has a maximum point at B and intersects the x-axis at the point C.

(i) Find the coordinates of A. [1]

(ii) Find the coordinates of B. [2]

(iii) Find the coordinates of C. [2]

(iv) Find $\int 2\cos\left(x - \frac{\pi}{6}\right)dx$. [1]

(v) Hence find the area of the shaded region. [2]

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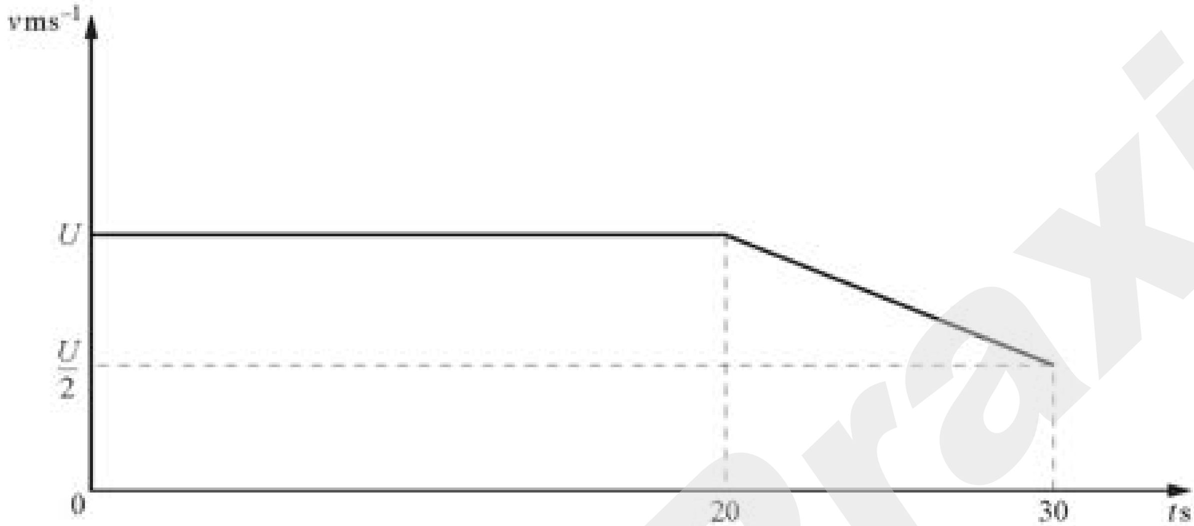
10.

(a)



The diagram shows part of the velocity-time graph for a particle, moving at $v\,\text{ms}^{-1}$ in a straight line, t s after passing through a fixed point. The particle travels at $U\,\text{ms}^{-1}$ for 20 s and then decelerates uniformly for 10 s to a velocity of $\frac{U}{2}\,\text{ms}^{-1}$. In this 30 s interval, the particle travels 165 m.

(i) Find the value of U. [3]

(ii) Find the acceleration of the particle between $t = 20$ and $t = 30$. [2]

(b) A particle P travels in a straight line such that, t s after passing through a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = \left(e^{\frac{t^2}{8}} - 4\right)^3$.

(i) Find the speed of P at O. [1]

(ii) Find the value of t for which P is instantaneously at rest. [2]

(iii) Find the acceleration of P when $t = 1$. [4]

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6.

(i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x^2 - 11))$. [2]

(ii) Hence show that $\int \frac{x}{3x^2 - 11}\,\mathrm{d}x = p\ln(3x^2 - 11) + c$, where p is a constant to be found, and c is a constant of integration. [1]

(iii) Given that $\int_2^a \frac{x}{3x^2 - 11}\,\mathrm{d}x = \ln 2$, where $a > 2$, find the value of a. [4]

10.

A curve $y = f(x)$ is such that $f'(x) = 6x - 8e^{2x}$.

(i) Given that the curve passes through the point $P\ (0, -3)$, find the equation of the curve. [5]

The normal to the curve $y = f(x)$ at P meets the line $y = 2 - 3x$ at the point Q.

(ii) Find the area of the triangle OPQ, where O is the origin. [5]

11.

A particle moving in a straight line has a velocity of v ms^{-1} such that, t s after leaving a fixed point, $v = 4t^2 - 8t + 3$.

(i) Find the acceleration of the particle when $t = 3$. [2]

(ii) Find the values of t for which the particle is momentarily at rest. [2]

(iii) Find the total distance the particle has travelled when $t = 1.5$. [5]

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1.

7.

The line $y = kx - 5$, where k is a positive constant, is a tangent to the curve $y = x^2 + 4x$ at the point A.

(i) Find the exact value of k. [3]

(ii) Find the gradient of the normal to the curve at the point A, giving your answer in the form $a + b\sqrt{5}$, where a and b are constants. [2]

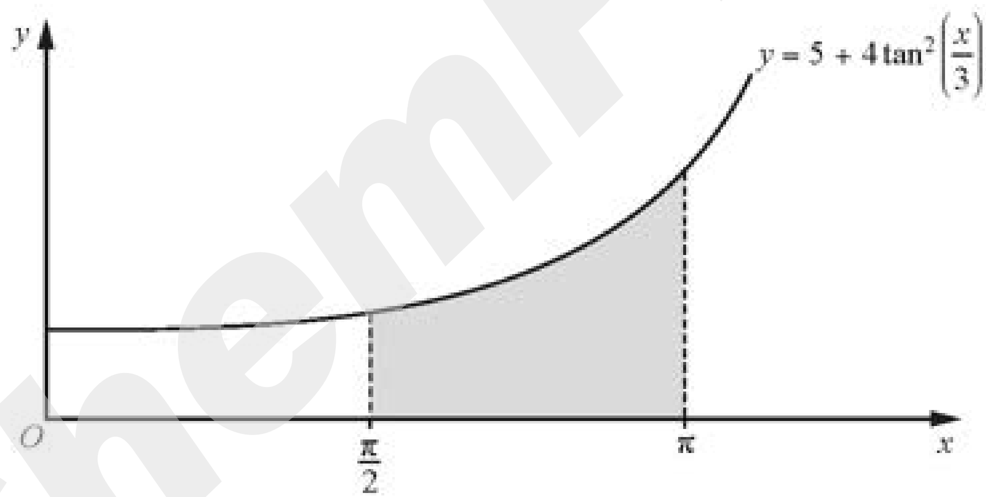
Show that the curve $y = (3x^2 + 8)^{\frac{5}{3}}$ has only one stationary point. Find the coordinates of this stationary point and determine its nature. [8]

9.

(i) Show that $5 + 4\tan^2\left(\frac{x}{3}\right) = 4\sec^2\left(\frac{x}{3}\right) + 1$. [1]

(ii) Given that $\frac{d}{dx}\left(\tan\left(\frac{x}{3}\right)\right) = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)$, find $\int \sec^2\left(\frac{x}{3}\right)dx$. [1]

(iii)



The diagram shows part of the curve $y = 5 + 4\tan^2\left(\frac{x}{3}\right)$. Using the results from parts **(i)** and **(ii)**, find the exact area of the shaded region enclosed by the curve, the x-axis and the lines $x = \frac{\pi}{2}$ and $x = \pi$. [5]

10.

(a) Given that $y = \dfrac{e^{3x}}{4x^2 + 1}$, find $\dfrac{dy}{dx}$. [3]

(b) Variables x, y and t are such that $y = 4\cos\left(x + \dfrac{\pi}{3}\right) + 2\sqrt{3}\sin\left(x + \dfrac{\pi}{3}\right)$ and $\dfrac{dy}{dt} = 10$.

(i) Find the value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{2}$. [3]

(ii) Find the value of $\dfrac{dx}{dt}$ when $x = \dfrac{\pi}{2}$. [2]

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2.

It is given that $y = \dfrac{(5x^2+4)^{\frac{1}{2}}}{x+1}$. Showing all your working, find the exact value of $\dfrac{dy}{dx}$ when $x = 3$. [5]

5.

A particle P moves in a straight line, such that its displacement, x m, from a fixed point O, t s after passing O, is given by $x = 4\cos(3t) - 4$.

(i) Find the velocity of P at time t. [1]

(ii) Hence write down the maximum speed of P. [1]

(iii) Find the smallest value of t for which the acceleration of P is zero. [3]

(iv) For the value of t found in part **(iii)**, find the distance of P from O. [1]

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6.

(i) Show that $\dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$. [4]

It is given that $\displaystyle\int_0^a \frac{\operatorname{cosec}2\theta}{\cot 2\theta + \tan 2\theta}\,\mathrm{d}\theta = \frac{\sqrt{3}}{4}$, where $0 < a < \dfrac{\pi}{4}$.

(ii) Using your answer to part **(i)** find the value of a, giving your answer in terms of π. [4]

11.

The curve $y = f(x)$ passes through the point $\left(\frac{1}{2}, \frac{7}{2}\right)$ and is such that $f'(x) = e^{2x-1}$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $f''(x) = 4$, giving your answer in the form $a + b\ln\sqrt{2}$, where a and b are constants. [4]

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9.

It is given that $\int_{-k}^{k}(15e^{5x} - 5e^{-5x})\,dx = 6$.

(i) Show that $e^{5k} - e^{-5k} = 3$. [5]

(ii) Hence, using the substitution $y = e^{5k}$, or otherwise, find the value of k. [3]

10.

It is given that $y = (10x + 2)\ln(5x + 1)$.

(i) Find $\frac{dy}{dx}$. [4]

(ii) Hence show that $\int \ln(5x+1)\,dx = \frac{(ax+b)}{5}\ln(5x+1) - x + c$, where a and b are integers and c is a constant of integration. [3]

(iii) Hence find $\int_0^{\frac{1}{5}} \ln(5x+1)\,dx$, giving your answer in the form $\frac{d + \ln f}{5}$, where d and f are integers. [2]

11.

A curve has equation $y = 6x - x\sqrt{x}$.

(i) Find the coordinates of the stationary point of the curve. [4]

(ii) Determine the nature of this stationary point. [2]

(iii) Find the approximate change in y when x increases from 4 to $4 + h$, where h is small. [3]

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12.

A particle moves in a straight line, such that its velocity, v ms^{-1}, t s after passing a fixed point O, is given by $v = 2 + 6t + 3\sin 2t$.

(i) Find the acceleration of the particle at time t. [2]

(ii) Hence find the smallest value of t for which the acceleration of the particle is zero. [2]

(iii) Find the displacement, x m from O, of the particle at time t. [5]

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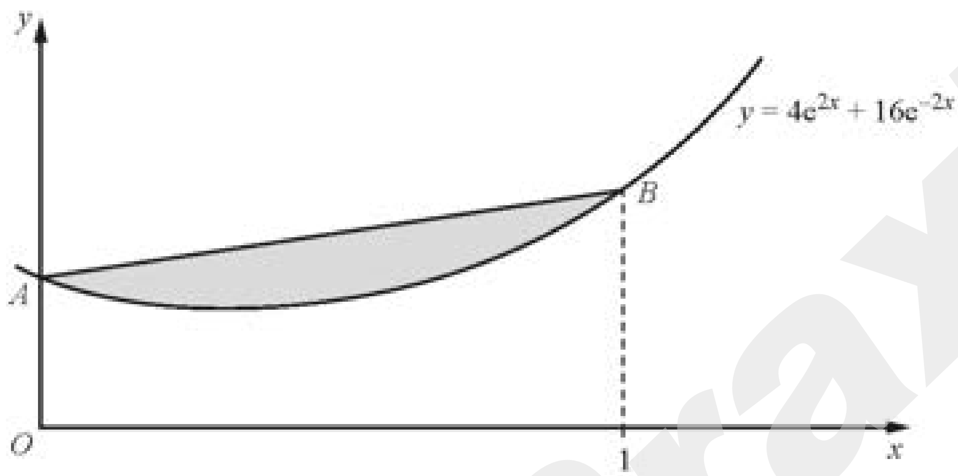
5.



The diagram shows part of the graph of $y = 4e^{2x} + 16e^{-2x}$ meeting the y-axis at the point A and the line $x = 1$ at the point B.

(i) Find the coordinates of A. [1]

(ii) Find the y-coordinate of B. [1]

(iii) Find $\int (4e^{2x} + 16e^{-2x})\,dx$. [2]

(iv) Hence find the area of the shaded region enclosed by the curve and the line AB. You must show all your working. [4]

7.

(i) Write $\ln\left(\frac{2x+1}{2x-1}\right)$ as the difference of two logarithms. [1]

A curve has equation $y = \ln\left(\frac{2x+1}{2x-1}\right) + 4x$ for $x > \frac{1}{2}$.

(ii) Using your answer to part **(i)** show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax^2+b}{4x^2-1}$, where a and b are integers. [4]

(iii) Hence find the x-coordinate of the stationary point on the curve. [2]

(iv) Determine the nature of this stationary point. [2]

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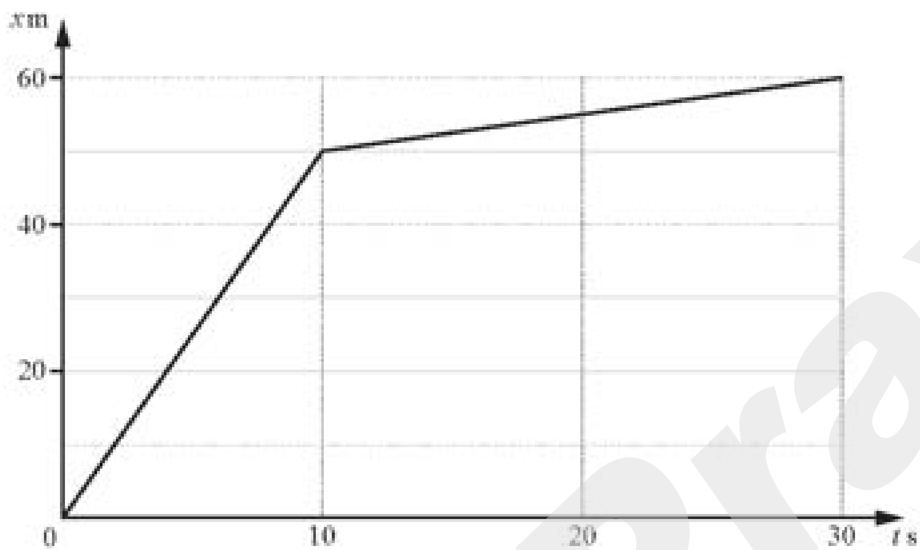
4.

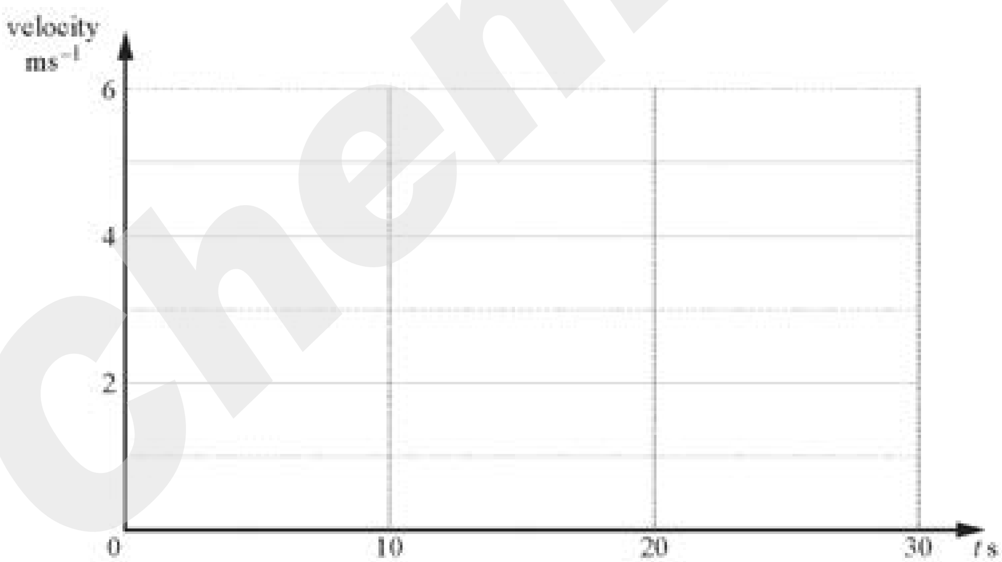
Given that $y = \frac{\ln(3x^2+2)}{x^2+1}$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2$, giving your answer as $a + b\ln 14$, where a and b are fractions in their simplest form. [6]

9.

(a)



The diagram shows the displacement-time graph of a particle P which moves in a straight line such that, t s after leaving a fixed point O, its displacement from O is x m. On the axes below, draw the velocity-time graph of P.



[3]

(b) A particle Q moves in a straight line such that its velocity, v ms^{-1}, t s after passing through a fixed point O, is given by $v = 3e^{-5t} + \frac{3t}{2}$, for $t \geqslant 0$.

(i) Find the velocity of Q when $t = 0$. [1]

(ii) Find the value of t when the acceleration of Q is zero. [3]

(iii) Find the distance of Q from O when $t = 0.5$. [4]

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2.

A curve is such that its gradient at the point (x, y) is given by $10e^{5x} + 3$. Given that the curve passes though the point (0, 9), find the equation of the curve. [4]

5.

(i) Find $\int (7x - 10)^{-\frac{1}{3}}\,dx$. [2]

(ii) Given that $\int_6^a (7x - 10)^{-\frac{1}{3}}\,dx = \frac{25}{14}$, find the exact value of a. [3]

8.

It is given that $y = (x-4)(3x-1)^{\frac{2}{3}}$.

(i) Show that $\dfrac{dy}{dx} = (3x-1)^{-\frac{1}{3}}(Ax+B)$, where A and B are integers to be found. [5]

(ii) Hence find, in terms of h, where h is small, the approximate change in y when x increases from 3 to $3+h$. [3]