Algebra and Graphs
(Past Year Topical Questions 2010-2015)

<u>May/June 2010 (41)</u>

8   **(a)** $f(x) = 2^x$

Complete the table.

x	-2	-1	0	1	2	3	4
$y = f(x)$		0.5	1	2	4		

[3]

(b) $g(x) = x(4 - x)$

Complete the table.

x	-1	0	1	2	3	4
$y = g(x)$		0	3		3	0

[2]

(c) On the grid, draw the graphs of

    (i) $y = f(x)$ for $-2 \leqslant x \leqslant 4$,   [3]

    (ii) $y = g(x)$ for $-1 \leqslant x \leqslant 4$.   [3]
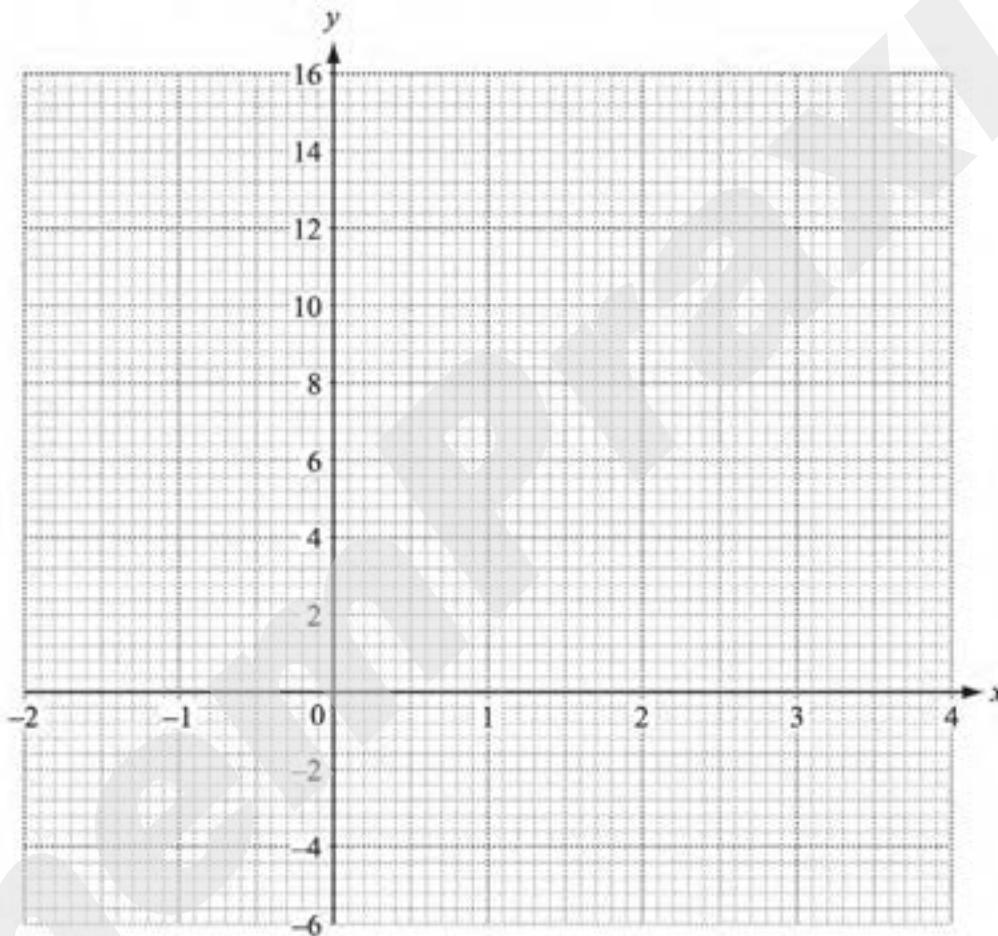
(d) Use your graphs to solve the following equations.

    (i) $f(x) = 10$

                                         Answer(d)(i) $x =$ .... [1]

(ii) $f(x) = g(x)$

Answer(d)(ii) $x =$ .... or $x =$ .... [2]

(iii) $f^{-1}(x) = 1.7$

Answer(d)(iii) $x =$ .... [1]

9 (a) Solve the following equations.

(i) $\dfrac{5}{w} = \dfrac{3}{w+1}$

Answer(a)(i) $w =$ .... [2]

(ii) $(y+1)^2 = 4$

Answer(a)(ii) $y =$ .... or $y =$ .... [2]

(iii) $\dfrac{x+1}{3} - \dfrac{x-2}{5} = 2$

Answer(a)(iii) $x =$ .... [3]

(b) (i) Factorise $u^2 - 9u - 10$.

Answer(b)(i) .... [2]

(ii) Solve the equation $u^2 - 9u - 10 = 0$.

Answer(b)(ii) $u =$ .... or $u =$ .... [1]

10 A company has a vehicle parking area of 1200 m^2 with space for x cars and y trucks.

Each car requires 20 m^2 of space and each truck requires 100 m^2 of space.

(a) Show that $x + 5y \leqslant 60$.

Answer(a)

[1]

(b) There must also be space for

    (i)   at least 40 vehicles,

    (ii) at least 2 trucks.

Write down two more inequalities to show this information.

Answer(b)(i) .... [1]

Answer(b)(ii) .... [1]

(c) One line has been drawn for you.
On the grid, show the three inequalities by drawing the other two lines and shading the **unwanted** regions.
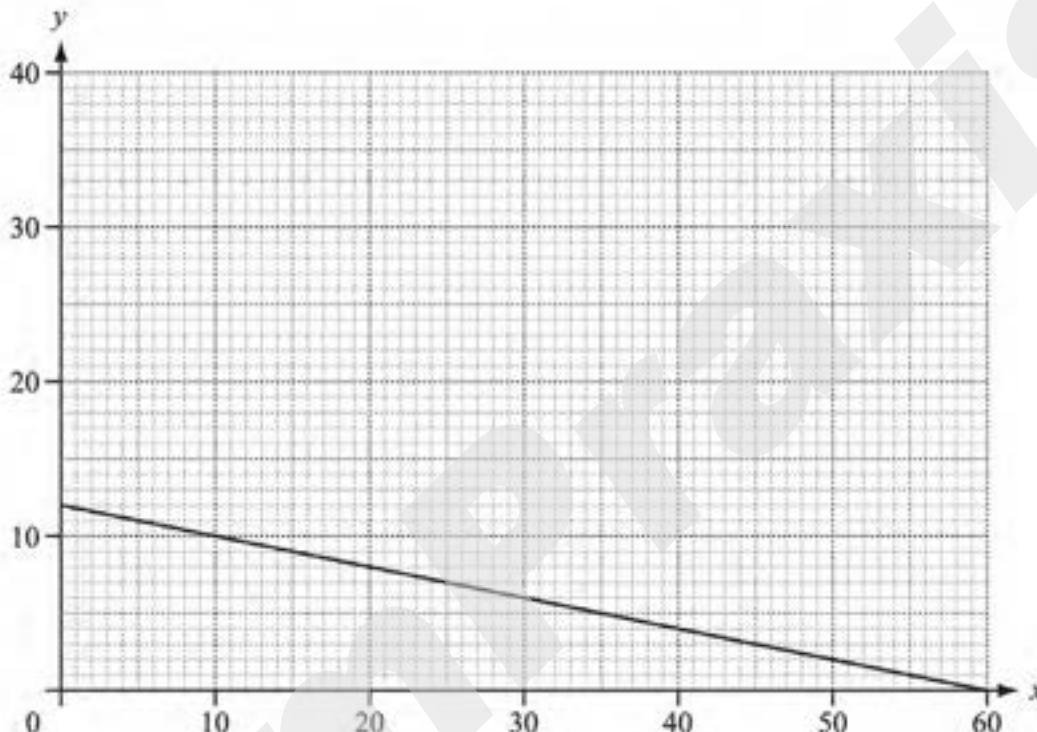
[4]

(d) Use your graph to find the largest possible number of trucks.

Answer(d) .... [1]

(e) The company charges $5 for parking each car and $10 for parking each truck.
Find the number of cars and the number of trucks which give the company the greatest possible income.

Calculate this income.

Answer(e) Number of cars = ...

Number of trucks = ...

Greatest possible income = $ .... [3]

11
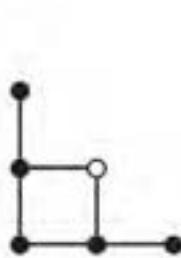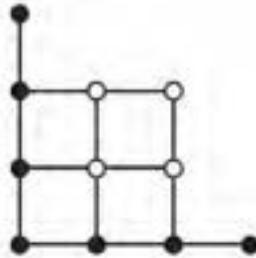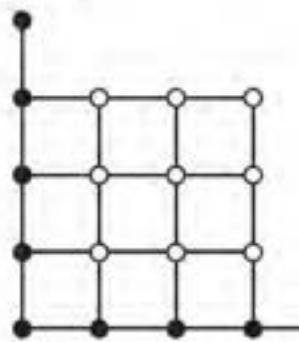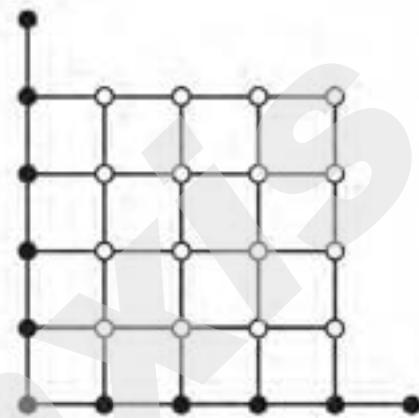
Diagram 1	Diagram 2	Diagram 3	Diagram 4
1 white dot	4 white dots	9 white dots	16 white dots
5 black dots	7 black dots	9 black dots	11 black dots
6 lines	14 lines	26 lines	42 lines

The four diagrams above are the first four of a pattern.

(a) Diagram 5 has been started below.
Complete this diagram and write down the information about the numbers of dots and lines.
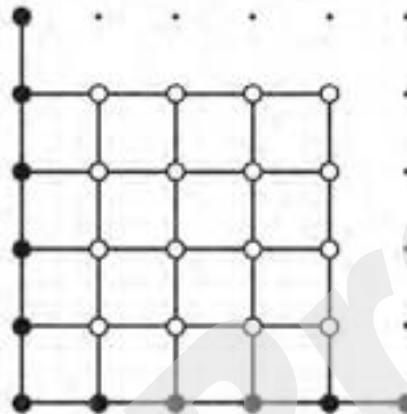
Diagram 5

.... white dots

.... black dots

.... lines

[4]

(b) Complete the information about the number of dots and lines in Diagram 8.

Answer(b) .... white dots

.... black dots

.... lines   [3]

9

(c) Complete the information about the number of dots in Diagram n.
Give your answers in terms of n.

Answer(c) .... white dots

.... black dots   [2]

(d) The number of lines in diagram n is $k(n^2 + n + 1)$.

Find

(i) the value of k,

Answer(d)(i) $k =$ .... [1]

(ii) the number of lines in Diagram 100.

Answer(d)(ii) .... [1]

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8 **(a)** y is 5 less than the square of the sum of p and q.

Write down a formula for y in terms of p and q.

Answer(a) $y =$ .... [2]

(b) The cost of a magazine is $\$x$ and the cost of a newspaper is $\$(x-3)$.

The total cost of 6 magazines and 9 newspapers is $51.

Write down and solve an equation in x to find the cost of a magazine.

Answer(b) $ .... [4]

(c) Bus tickets cost $3 for an adult and $2 for a child.

There are a adults and c children on a bus.

The total number of people on the bus is 52.

The total cost of the 52 tickets is $139.

Find the number of adults and the number of children on the bus.

Answer(c) Number of adults = ...

Number of children = ... [5]

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6 (a) Complete the table of values for $y = x + \dfrac{1}{x}$.

x	-4	-3	-2	-1	-0.5	0.5	1	2	3	4
y	-4.3	-3.3			-2.5	2.5			3.3	4.3

[2]

(b)
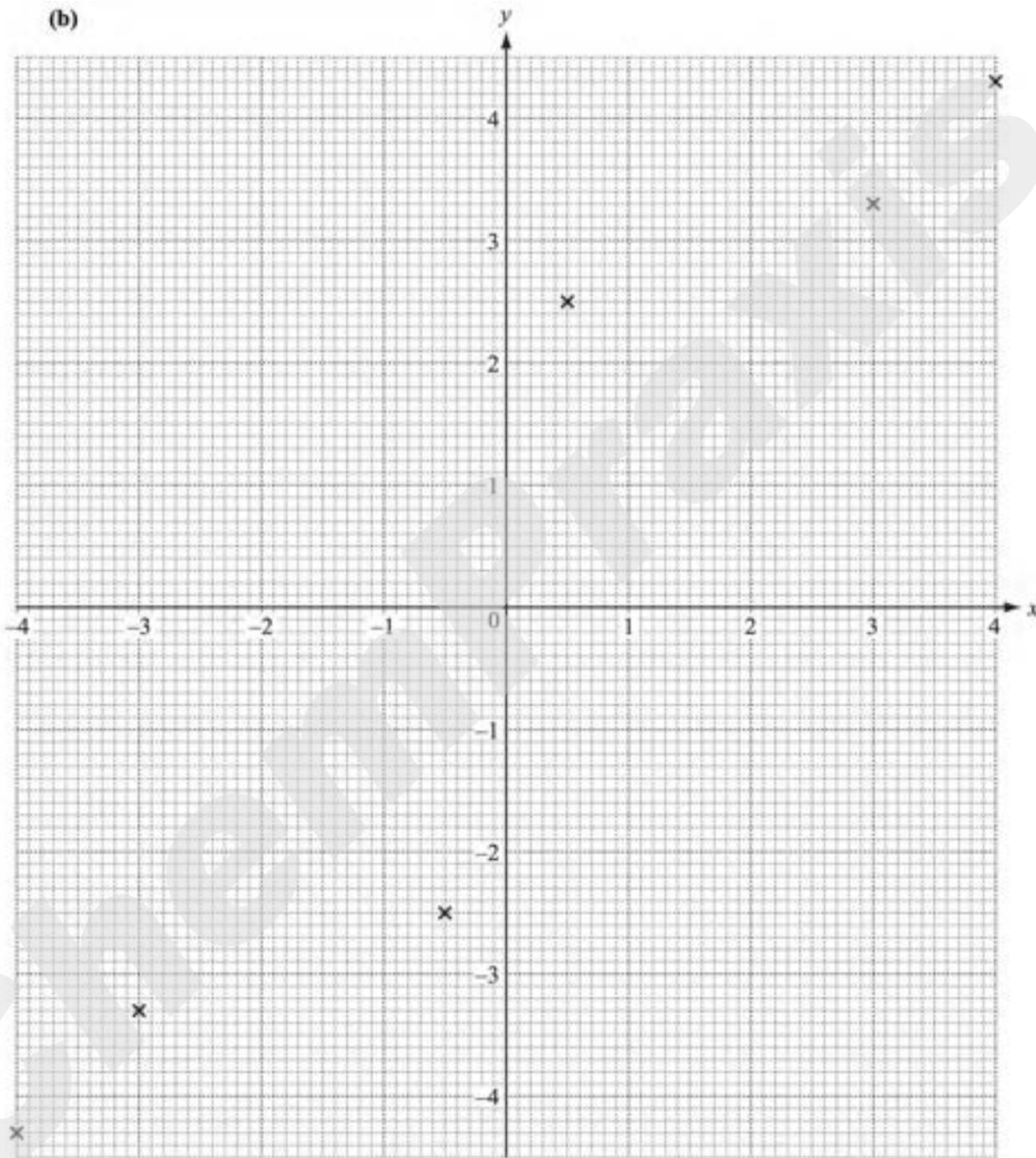
On the grid, draw the graph of $y = x + \dfrac{1}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

Six of the ten points have been plotted for you.

[3]

(c) There are three integer values of k for which the equation $x + \dfrac{1}{x} = k$ has **no** solutions.

Write down these three values of k.

Answer(c) $k =$ .... or $k =$ .... or $k =$ .... [2]

(d) Write down the ranges of x for which the gradient of the graph of $y = x + \dfrac{1}{x}$ is positive.

Answer(d) .... [2]

(e) To solve the equation $x + \dfrac{1}{x} = 2x + 1$, a straight line can be drawn on the grid.

(i) Draw this line on the grid for $-2.5 \leqslant x \leqslant 1.5$. [2]

(ii) On the grid, show how you would find the solutions. [1]

(iii) Show how the equation $x + \dfrac{1}{x} = 2x + 1$ can be rearranged into the form $x^2 + bx + c = 0$

and find the values of b and c.

Answer(e)(iii) $b =$ ....

$c =$ .... [3]

9   (a)   The cost of a bottle of water is $w.

The cost of a bottle of juice is $j.

The total cost of 8 bottles of water and 2 bottles of juice is $12.

The total cost of 12 bottles of water and 18 bottles of juice is $45.

Find the cost of a bottle of water and the cost of a bottle of juice.

Answer(a) Cost of a bottle of water = $ ....

Cost of a bottle of juice = $ ....   [5]

(b) Roshni cycles 2 kilometres at y km/h and then runs 4 kilometres at $(y-4)$ km/h.
The whole journey takes 40 **minutes**.

(i) Write an equation in y and show that it simplifies to $y^2 - 13y + 12 = 0$.

Answer(b)(i)

[4]

(ii) Factorise $y^2 - 13y + 12$.

Answer(b)(ii) .... [2]

(iii) Solve the equation $y^2 - 13y + 12 = 0$.

Answer(b)(iii) $y = $ .... or $y = $ .... [1]

(iv) Work out Roshni's running speed.

Answer(b)(iv) .... km/h [1]

(c) Solve the equation

$$u^2 - u - 4 = 0.$$

Show all your working and give your answers correct to 2 decimal places.

Answer(c) u = ... or *u =* .... [4]

10
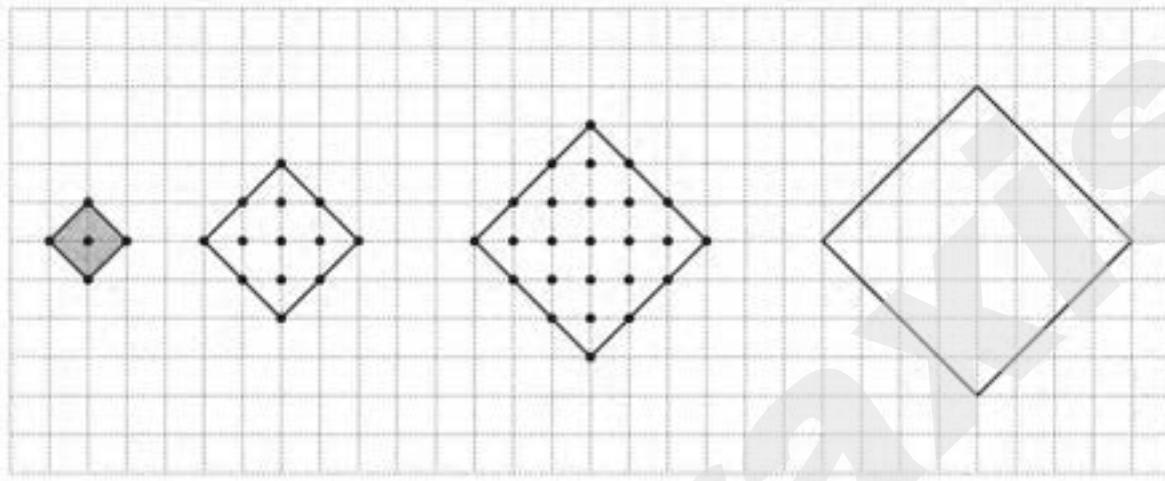
Diagram 1   Diagram 2   Diagram 3   Diagram 4

The diagrams show squares and dots on a grid.

Some dots are on the sides of each square and other dots are inside each square.

The area of the square (shaded) in Diagram 1 is 1 unit^2.

(a) Complete Diagram 4 by marking all the dots. [1]

(b) Complete the columns in the table below for Diagrams 4, 5 and n.

Diagram	1	2	3	4	5	--------	n
Number of units of area	1	4	9			--------	
Number of dots inside the square	1	5	13			--------	$(n-1)^2 + n^2$
Number of dots on the sides of the square	4	8	12			--------	
Total number of dots	5	13	25			--------	

[7]

(c) For Diagram 200, find the number of dots

    (i) inside the square,

Answer(c)(i) .... [1]

    (ii) on the sides of the square.

Answer(c)(ii) .... [1]

(d) Which diagram has 265 dots inside the square?

Answer(d) .... [1]

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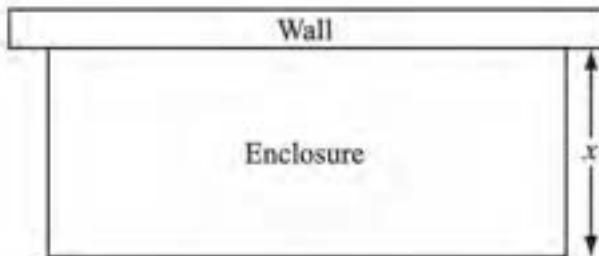
A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width x metres and area A square metres.

(a) Show that $A = 72x - 2x^2$.

Answer (a)

[2]

(b) Factorise completely $72x - 2x^2$.

Answer(b) .... [2]

(c) Complete the table for $A = 72x - 2x^2$.

x	0	5	10	15	20	25	30	35
A	0	310	520			550	360	

[3]

(d) Draw the graph of $A = 72x - 2x^2$ for $0 \leqslant x \leqslant 35$ on the grid opposite.
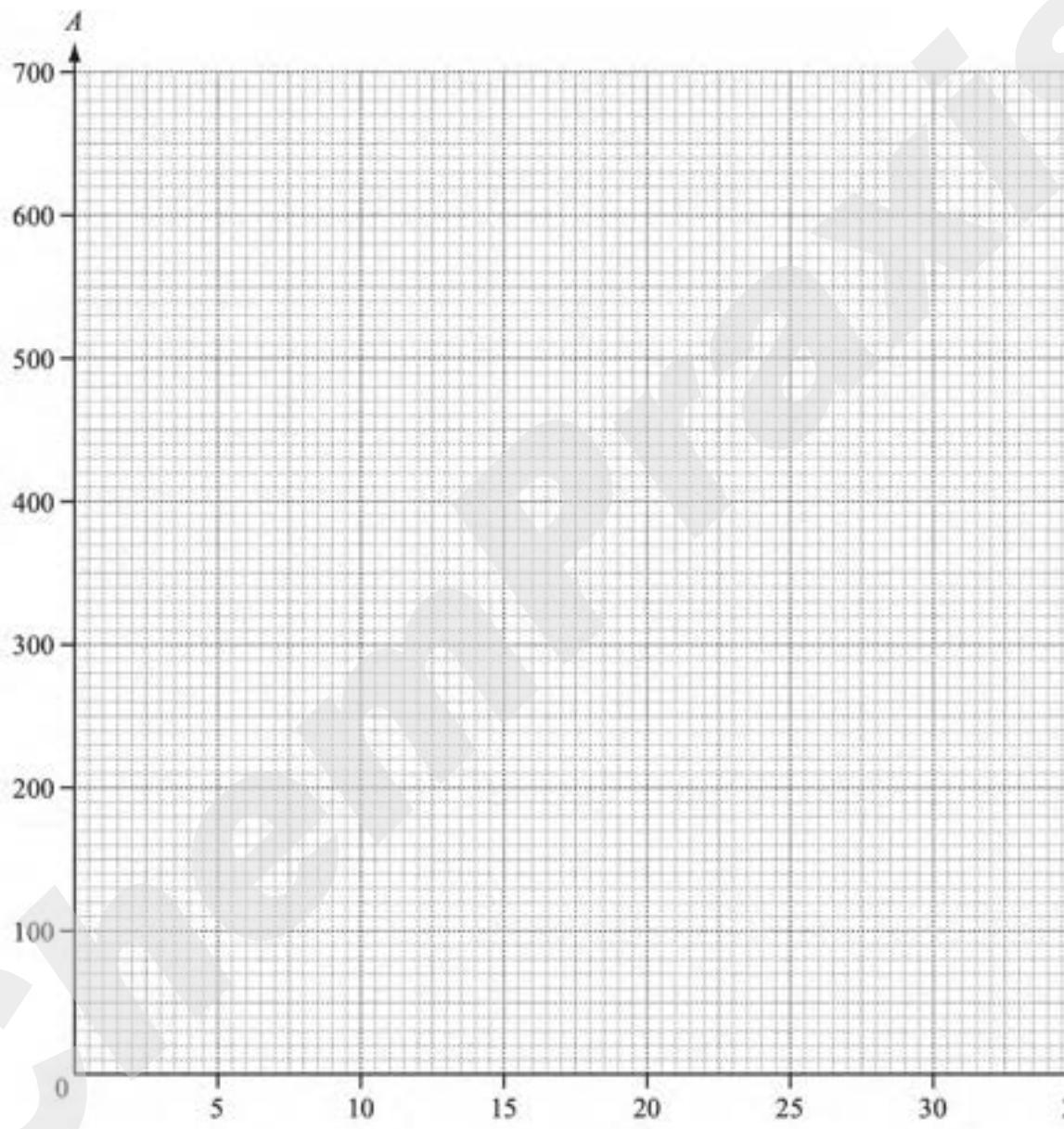
[4]

(e) Use your graph to find

(i) the values of x when $A = 450$,

Answer(e)(i) $x =$ .... or $x =$ .... [2]

(ii) the maximum area of the enclosure.

Answer(e)(ii) .... m^2 [1]

(f) Each animal must have at least 12 m^2 for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of 500 m^2.

Answer(f) .... [2]

8 (a) $f(x) = 2x - 1$ $\qquad$ $g(x) = x^2$

Work out

(i) $f(2)$,

Answer(a)(i) .... [1]

(ii) $g(-2)$,

Answer(a)(ii) .... [1]

(iii) $ff(x)$ in its simplest form,

Answer(a)(iii) $ff(x) =$ .... [2]

(iv) $f^{-1}(x)$, the inverse of $f(x)$,

Answer(a)(iv) $f^{-1}(x) =$ .... [2]

(v) x when $gf(x) = 4$.

Answer(a)(v) $x =$ .... or $x =$ .... [4]

(b) y is **inversely** proportional to x and $y = 8$ when $x = 2$.

Find,

(i) an equation connecting y and x,

Answer(b)(i) .... [2]

(ii) y when $x = \dfrac{1}{2}$.

Answer(b)(ii) $y =$ .... [1]

9   (a)   The first five terms P_1, P_2, P_3, P_4 and P_5 of a sequence are given below.

$$1 \qquad\qquad = 1 = P_1$$

$$1 + 2 \qquad\qquad = 3 = P_2$$

$$1 + 2 + 3 \qquad\qquad = 6 = P_3$$

$$1 + 2 + 3 + 4 \qquad\qquad = 10 = P_4$$

$$1 + 2 + 3 + 4 + 5 \qquad = 15 = P_5$$

    (i)   Write down the next term, P_6, in the sequence 1, 3, 6, 10, 15...

Answer(a)(i) .... [1]

    (ii)   The formula for the nth term of this sequence is

$$P_n = \frac{1}{2}n(n + 1).$$

Show this formula is true when $n = 6$.

Answer (a)(ii)

[1]

    (iii)   Use the formula to find P_{50}, the 50th term of this sequence.

Answer(a)(iii) .... [1]

    (iv)   Use your answer to **part (iii)** to find $3 + 6 + 9 + 12 + 15 + \ldots\ldots\ldots + 150$.

Answer(a)(iv) .... [1]

(vi) Use your answers to **parts (iv)** and **(v)** to find the sum of the numbers less than 150 which are **not** multiples of 3.

Answer(a)(vi) .... [1]

(b) The first five terms, S_1, S_2, S_3, S_4 and S_5 of a different sequence are given below.

(1×1) $= 1 = S_1$

$(1 \times 2) + (2 \times 1)$ $= 4 = S_2$

$(1 \times 3) + (2 \times 2) + (3 \times 1)$ $= 10 = S_3$

$(1 \times 4) + (2 \times 3) + (3 \times 2) + (4 \times 1)$ $= 20 = S_4$

$(1 \times 5) + (2 \times 4) + (3 \times 3) + (4 \times 2) + (5 \times 1)$ $= 35 = S_5$

(i) Work out the next term, S_6, in the sequence 1, 4, 10, 20, 35…

Answer(b)(i) .... [2]

(ii) The formula for the nth term of this sequence is

$$S_n = \frac{1}{6}n(n + 1)(n + 2).$$

Show this formula is true for $n = 6$.

Answer(b)(ii)

[1]

(iii) Find $(1 \times 20) + (2 \times 19) + (3 \times 18) \ldots\ldots\ldots + (20 \times 1)$.

Answer(b)(iii) .... [1]

(c) Show that $S_6 - S_5 = P_6$, where P_6 is your answer to **part (a)(i)**.

Answer(c)

[1]

(d) Show by algebra that $S_n - S_{n-1} = P_n$. $[P_n = \frac{1}{2}n(n+1)]$

Answer(d)

[3]

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2 $f(x) = 6 + x^2$ $g(x) = 4x - 1$

(a) Find

   (i) g(3),

Answer(a)(i) .... [1]

   (ii) $f(-4)$.

Answer(a)(ii) .... [1]

(b) Find the inverse function $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ .... [2]

(c) Find $fg(x)$ in its simplest form.

Answer(c) $fg(x) =$ .... [3]

(d) Solve the equation $gg(x) = 3$.

Answer(d) $x =$ .... [3]

7   (a) Complete the table for the function $f(x) = \dfrac{2}{x} - x^2$.

x	-3	-2	-1	-0.5	-0.2		0.2	0.5	1	2	3
$f(x)$	-9.7	-5			-10.0		10.0	3.75	1		-8.3

[3]

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.

[5]
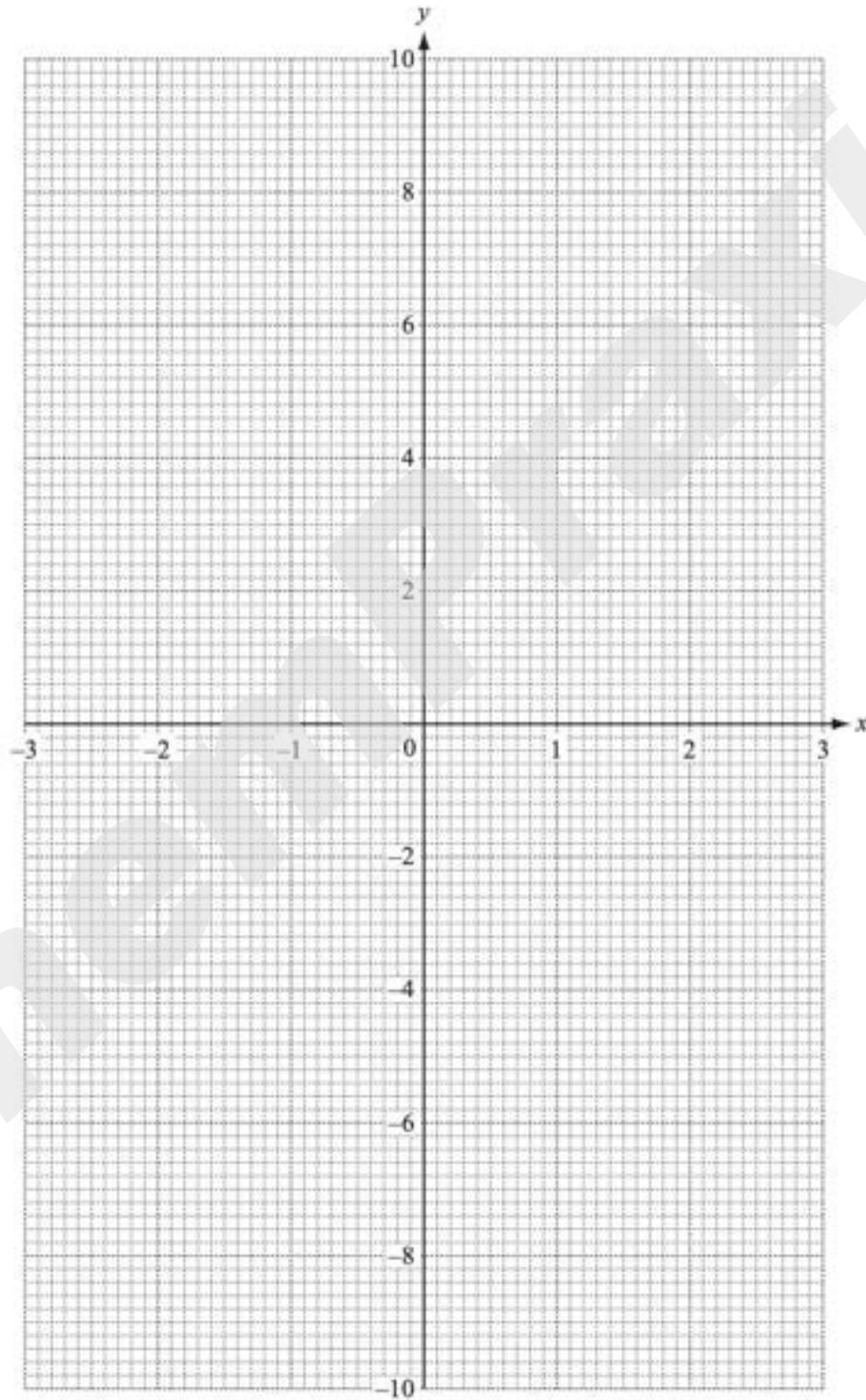
(c) Use your graph to

(i) solve f(x) = 2,

Answer(c)(i) $x =$ .... [1]

(ii) find a value for k so that f(x) = k has 3 solutions.

Answer(c)(ii) $k =$ .... [1]

(d) Draw a suitable line on the grid and use your graphs to solve the equation $\dfrac{2}{x} - x^2 = 5x$.

Answer(d) $x =$ .... or $x =$ .... [3]

(e) Draw the tangent to the graph of y = f(x) at the point where $x = -2$.

Use it to calculate an estimate of the gradient of y = f(x) when $x = -2$.

Answer(e) .... [3]

10 In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.

(a) Write down the next two terms in this sequence.

     1   1   2   3   5   8   13   ....   [1]

(b) Write down the first two terms of this sequence.

       3   11   14   [2]

(c) (i) Find the value of d and the value of e.

     2   d e 10

Answer(c)(i) d = ...

               e = ... [3]

(ii) Find the value of x, the value of y and the value of z.

$$-33 \qquad x \qquad y \qquad z \qquad 18$$

Answer(c)(ii) $x = $ ....

$y = $ ....

$z = $ .... [5]

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3 **(a)** Expand the brackets and simplify.

$$x(x+3)+4x(x-1)$$

Answer(a) .... [2]

(b) Simplify $(3x^3)^3$.

Answer(b) .... [2]

(c) Factorise the following completely.

(i) $7x^7 + 14x^{14}$

Answer(c)(i) .... [2]

(ii) $xy + xw + 2ay + 2aw$

Answer(c)(ii) .... [2]

(iii) $4x^2 - 49$

Answer(c)(iii) .... [1]

(d) Solve the equation.

$$2x^2 + 5x + 1 = 0$$

Show all your working and give your answers correct to 2 decimal places.

*Answer(d) x =* .... or *x =* .... [4]

7 **(a)** Complete the table for the function $f(x) = \dfrac{x^3}{10} + 1$.

x	−4	−3	−2	−1	0	1	2	3
f(x)		−1.7	0.2	0.9	1	1.1	1.8	

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 3$.
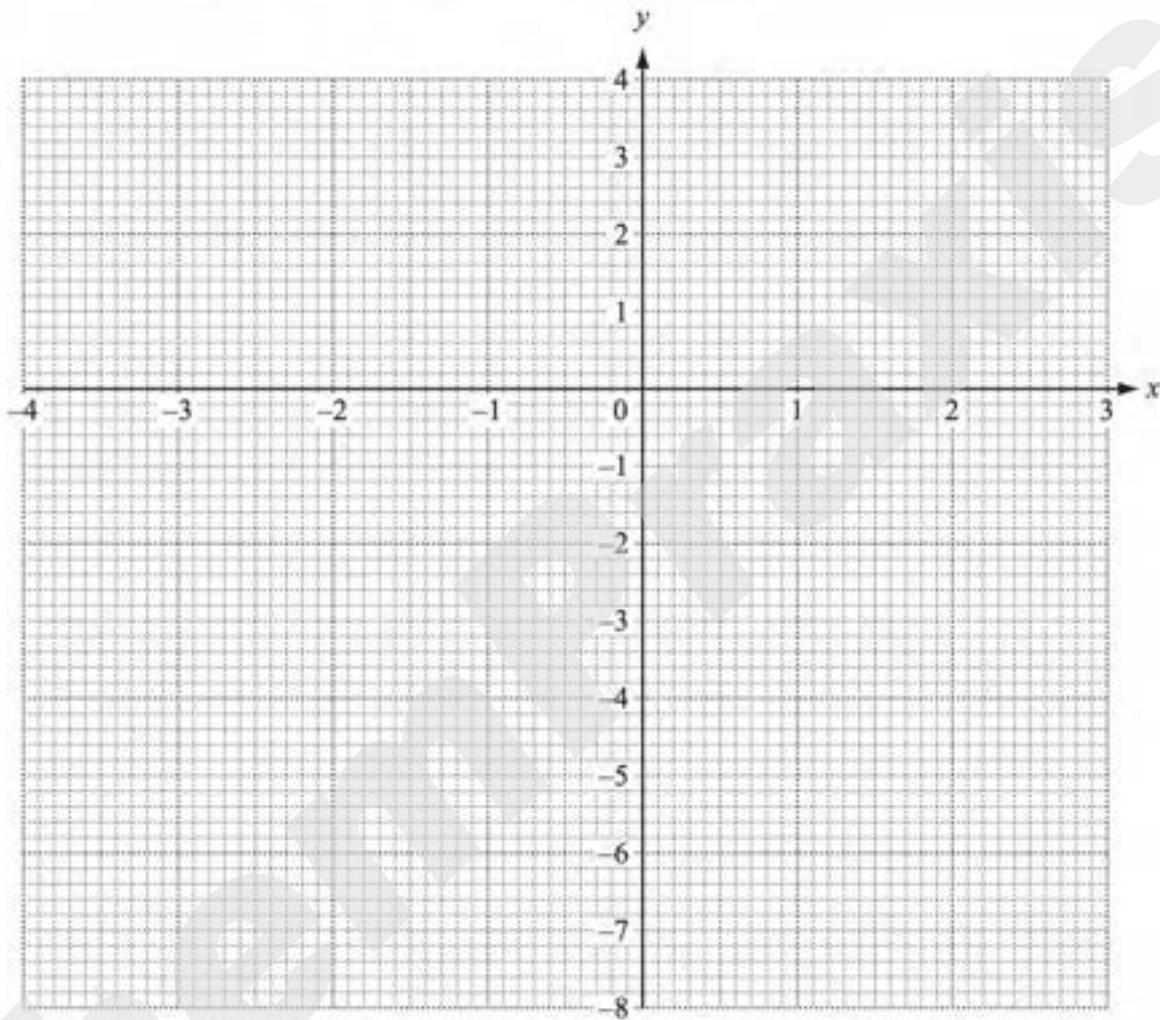
[4]

(c) Complete the table for the function $g(x) = \dfrac{4}{x}$, $x \neq 0$.

x	-4	-3	-2	-1	1	2	3
$g(x)$	-1	-1.3				2	1.3

[2]

(d) On the grid, draw the graph of $y = g(x)$ for $-4 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 3$. [3]

(e) (i) Use your graphs to solve the equation $\dfrac{x^3}{10} + 1 = \dfrac{4}{x}$.

Answer(e)(i) $x =$ .... or $x =$ .... [2]

(ii) The equation $\dfrac{x^3}{10} + 1 = \dfrac{4}{x}$ can be written as $x^4 + ax + b = 0$.

Find the values of a and b.

Answer(e)(ii) $a =$ ....

$b =$ .... [2]

11

| Diagram 1 | Diagram 2 | Diagram 3 | Diagram 4 |
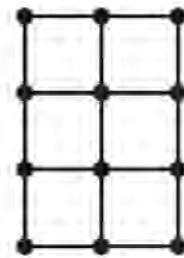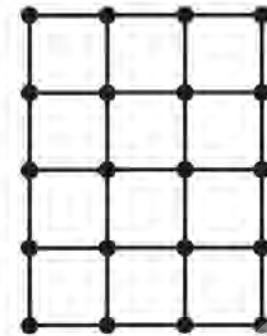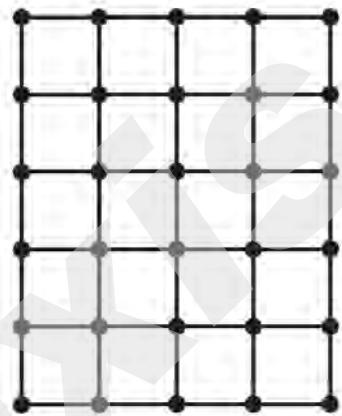
The first four Diagrams in a sequence are shown above.
Each Diagram is made from dots and one centimetre lines.
The area of each small square is 1 cm².

(a) Complete the table for Diagrams 5 and 6.

Diagram	1	2	3	4	5	6
Area (cm²)	2	6	12	20		
Number of dots	6	12	20	30		
Number of one centimetre lines	7	17	31	49		

[4]

(b) The **area** of Diagram n is $n(n+1)$ cm².

(i) Find the **area** of Diagram 50.

Answer(b)(i) .... cm² [1]

(ii) Which Diagram has an **area** of 930 cm^2?

Answer(b)(ii) .... [1]

(c) Find, in terms of n, the number of **dots** in Diagram n.

Answer(c) .... [1]

(d) The number of one centimetre lines in Diagram n is $2n^2 + pn + 1$.

(i) Show that $p = 4$.

Answer(d)(i)

[2]

(ii) Find the number of one centimetre lines in Diagram 10.

Answer(d)(ii) .... [1]

(iii) Which Diagram has 337 one centimetre lines?

Answer(d)(iii) .... [3]

(e) For **each** Diagram, the number of squares of area 1 cm^2 is A, the number of dots is D and the number of one centimetre lines is L.

Find a connection between A, D and L that is true for each Diagram.

Answer(e) .... [1]

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3   (a) p varies **inversely** as $(m + 1)$.

   When $p = 4$, $m = 8$.

   Find the value of p when $m = 11$.

Answer(a) $p =$ .... [3]

(b)   (i)   Factorise $x^2 - 25$.

Answer(b)(i) .... [1]

   (ii)   Simplify $\dfrac{2x^2 + 11x + 5}{x^2 - 25}$.

Answer(b)(ii) .... [3]

(c) Solve the inequality $5(x-4) < 3(12-x)$.

Answer(c) .... [3]

7 (a) Complete the table of values for the equation $y = \dfrac{4}{x^2}$, $x \neq 0$.

x	-4	-3	-2	-1	-0.6		0.6	1	2	3	4
y	0.25	0.44			11.11			4.00		0.44	

[3]

(b) On the grid, draw the graph of $y = \dfrac{4}{x^2}$ for $-4 \leqslant x \leqslant -0.6$ and $0.6 \leqslant x \leqslant 4$.

[5]
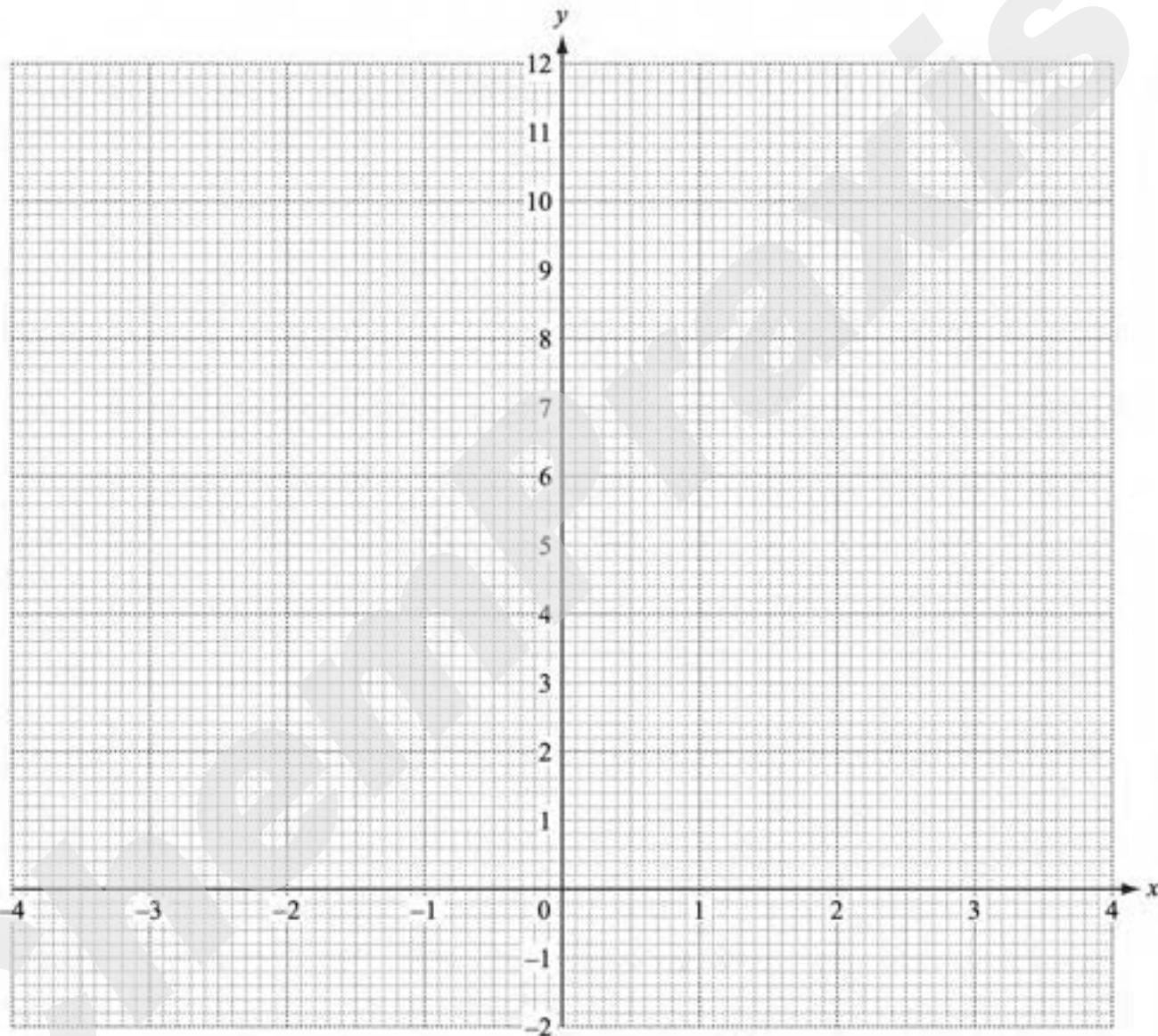
(c) Use your graph to solve the equation $\dfrac{4}{x^2} = 6$.

Answer(c)x = ... or *x =* .... [2]

(d) By drawing a suitable tangent, estimate the gradient of the graph where $x = 1.5$.

Answer(d) .... [3]

(e) (i) The equation $\dfrac{4}{x^2} - x + 2 = 0$ can be solved by finding the intersection of the graph

of $y = \dfrac{4}{x^2}$ and a straight line.

Write down the equation of this straight line.

Answer(e)(i) .... [1]

(ii) On the grid, draw the straight line from your answer to **part (e)(i)**. [2]

(iii) Use your graphs to solve the equation $\dfrac{4}{x^2} - x + 2 = 0$.

Answer(e)(iii) x = ... [1]

9 Peter wants to plant x plum trees and y apple trees.

He wants at least 3 plum trees and at least 2 apple trees.

(a) Write down one inequality in x and one inequality in y to represent these conditions.

Answer(a) .... , ... [2]

(b) There is space on his land for no more than 9 trees.

Write down an inequality in x and y to represent this condition.

Answer(b) .... [1]

(c) Plum trees cost $6 and apple trees cost $14.

Peter wants to spend no more than $84.

Write down an inequality in x and y, and show that it simplifies to $3x + 7y \leqslant 42$.

Answer(c)

[1]

(d) On the grid, draw four lines to show the four inequalities and shade the **unwanted** regions.
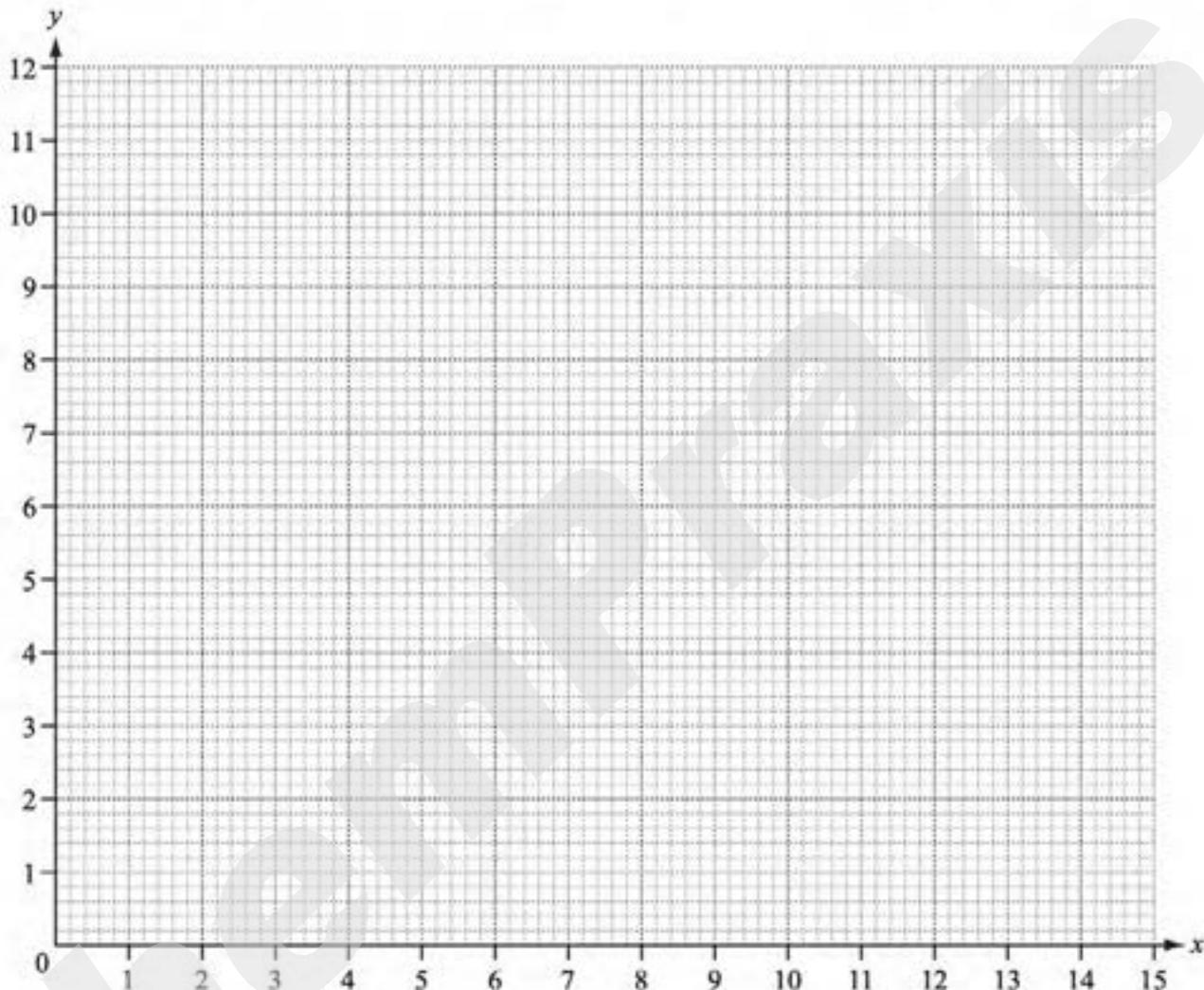
[7]

(e) Calculate the smallest cost when Peter buys a total of 9 trees.

Answer(e) $ .... [2]

10 The first and the nth terms of sequences A, B and C are shown in the table below.

(a) Complete the table for each sequence.

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	1					n^3
Sequence B	4					$4n$
Sequence C	4					$(n+1)^2$

[5]

(b) Find

(i) the 8th term of sequence A,

Answer(b)(i) .... [1]

(ii) the 12th term of sequence C.

Answer(b)(ii) .... [1]

(c) (i) Which term in sequence A is equal to $15\,625$?

Answer(c)(i) .... [1]

(ii) Which term in sequence C is equal to $10\,000$?

Answer(c)(ii) .... [1]

(d) The first four terms of sequences D and E are shown in the table below.

Use the results from **part (a)** to find the 5th and the nth terms of the sequences D and E.

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80		
Sequence E	0	1	4	9		

[4]

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4 (a) Complete the table of values for the function $y = x^2 - \dfrac{3}{x}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.25		0.25	0.5	1	2	3
y	10	5.5		6.3	12.1		-11.9			2.5	8

[3]

(b) Draw the graph of $y = x^2 - \dfrac{3}{x}$ for $-3 \leqslant x \leqslant -0.25$ and $0.25 \leqslant x \leqslant 3$.

[5]
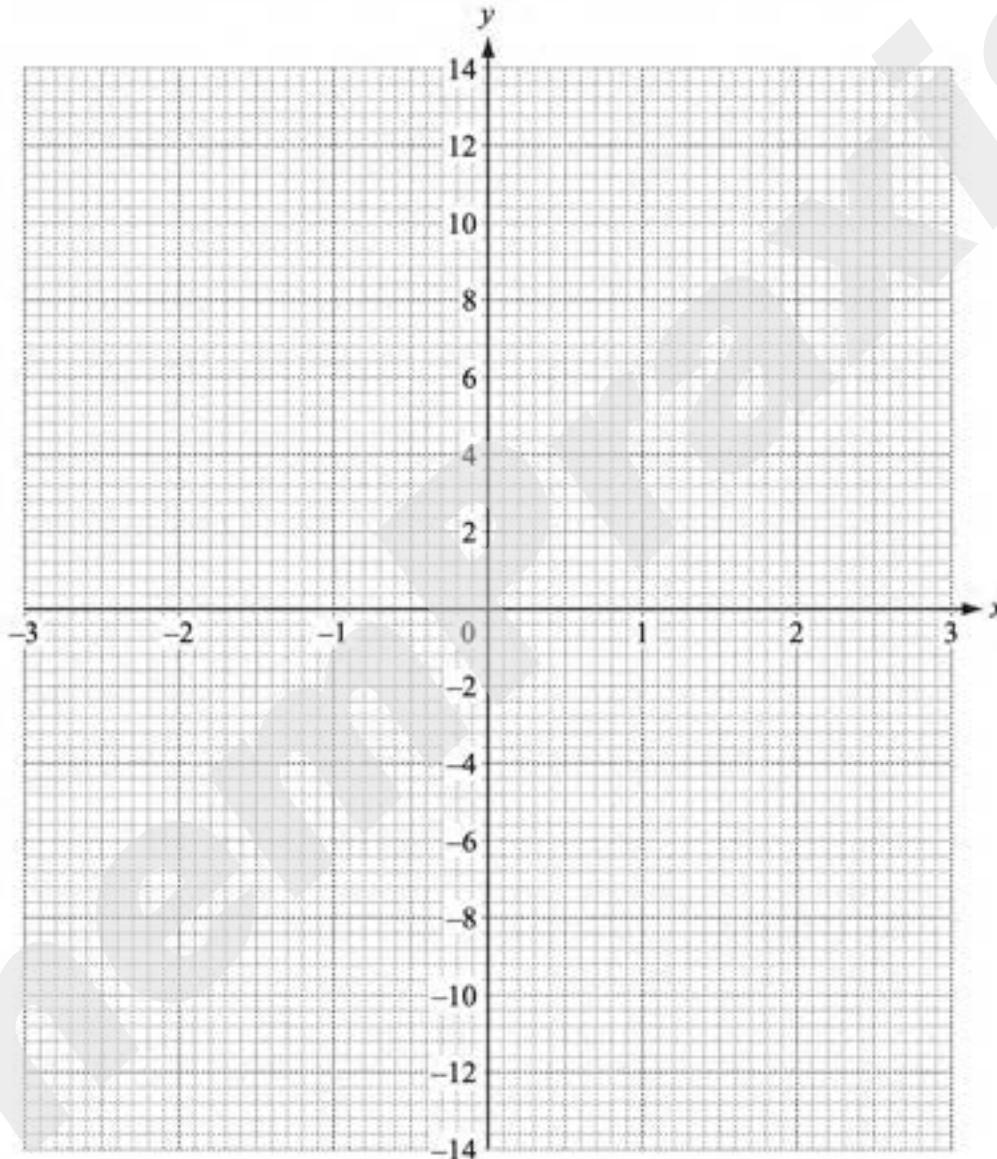
(c) Use your graph to solve $x^2 - \dfrac{3}{x} = 7$.

Answer(c) x = ... or *x* = ... or *x* = ... [3]

(d) Draw the tangent to the curve where $x = -2$.
Use the tangent to calculate an estimate of the gradient of the curve where $x = -2$.

Answer(d) .... [3]

5 **(a)** Solve $9 < 3n + 6 \leqslant 21$ for integer values of n.

Answer(a) .... [3]

(b) Factorise completely.

(i) $2x^2 + 10xy$

Answer(b)(i) .... [2]

(ii) $3a^2 - 12b^2$

Answer(b)(ii) .... [3]

(d) Solve

$$\frac{15-x}{2} = 3 - 2x.$$

Answer(d) x = ... [3]

(e) Solve $2x^2 - 5x - 6 = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(e) x = ... *or x =* .... [4]

9   **(a)**   **(i)**   Work out the first 3 terms of the sequence whose nth term is $n(n+2)$.

                                                 Answer(a)(i)   .... , ... , ...   [2]

      (ii)   Which term in this sequence is equal to 168?

                                                 Answer(a)(ii)   ....   [3]

  (b)   Find a formula for the nth term of the following sequences.

      (i)   5   8   11   14   17 ......

                                              Answer(b)(i)   ....   [2]

      (ii)   1   2   4   8   16 ......

                                              Answer(b)(ii)   ....   [2]

(c)

| Diagram 1 | Diagram 2 | Diagram 3 |
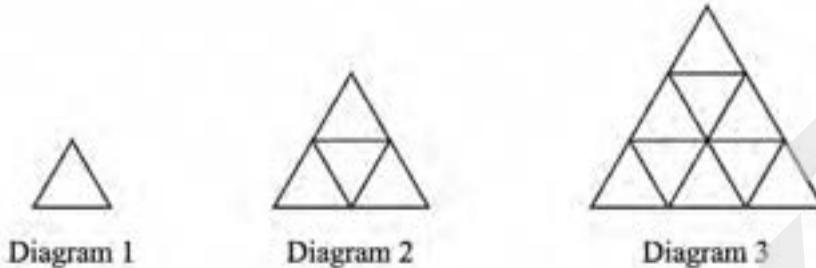
A sequence of diagrams is formed by drawing equilateral triangles each of side one centimetre.
Diagram 1 has 3 one centimetre lines.
Diagram 2 has 9 one centimetre lines.

The formula for the **total** number of one centimetre lines needed to draw **all of the first n diagrams** is

$$an^3 + bn^2 + n.$$

Find the values of a and b.

Answer(c) $a =$ ....

$b =$ .... [6]

<u>May/June 2011 (43)</u>

5 **(a)** Complete the table of values for the function $f(x)$, where $f(x) = x^2 + \dfrac{1}{x^2}$, $x \neq 0$.

x	-3	-2.5	-2	-1.5	-1	-0.5		0.5	1	1.5	2	2.5	3
$f(x)$		6.41		2.69		4.25		4.25		2.69		6.41	

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$.
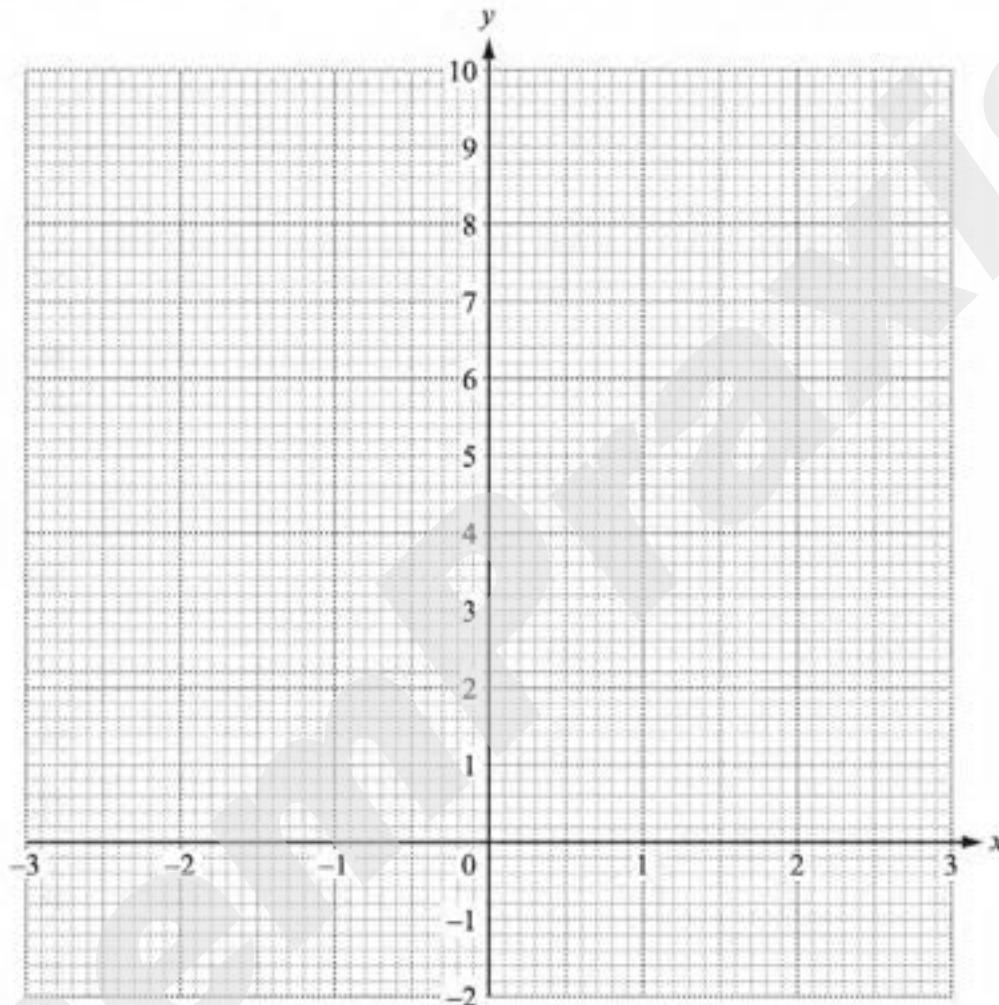
[5]

(c) (i) Write down the equation of the line of symmetry of the graph.

Answer(c)(i) .... [1]

(ii) Draw the tangent to the graph of $y = f(x)$ where $x = -1.5$.
Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

Answer(c)(ii) .... [3]

(iii) Use your graph to solve the equation $x^2 + \dfrac{1}{x^2} = 3$.

Answer(c)(iii) $x =$ .... or $x =$ .... or $x =$ .... or $x =$ .... [2]

(iv) Draw a suitable line on the grid and use your graphs to solve the equation $x^2 + \dfrac{1}{x^2} = 2x$.

Answer(c)(iv) $x =$ .... or $x =$ .... [3]

9   $f(x) = 3x + 1$ $g(x) = (x + 2)^2$

(a) Find the values of

(i) gf(2),

Answer(a)(i) .... [2]

(ii) ff(0.5).

Answer(a)(ii) .... [2]

(b) Find $f^{-1}(x)$, the inverse of $f(x)$.

Answer(b) .... [2]

(c) Find $fg(x)$.

Give your answer in its simplest form.

Answer(c) .... [2]

(d) Solve the equation $x^2 + f(x) = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(d) x = ... or *x =* .... [4]

11 (a) (i) The first three positive integers 1, 2 and 3 have a sum of 6.

Write down the sum of the first 4 positive integers.

Answer(a)(i) .... [1]

(ii) The formula for the sum of the first n integers is $\dfrac{n(n+1)}{2}$.

Show the formula is correct when $n = 3$.

Answer(a)(ii)

[1]

(iii) Find the sum of the first 120 positive integers.

Answer(a)(iii) .... [1]

(iv) Find the sum of the integers

$121 + 122 + 123 + 124 +$ .... $+ 199 + 200.$

Answer(a)(iv) .... [2]

(v) Find the sum of the even numbers

$2 + 4 + 6 +$ .... $+ 800.$

Answer(a)(v) .... [2]

(b) (i) Complete the following statements about the sums of cubes and the sums of integers.

$1^3 = 1$	$1 = 1$
$1^3 + 2^3 = 9$	$1 + 2 = 3$
$1^3 + 2^3 + 3^3 = $	$1 + 2 + 3 = $ ....
$1^3 + 2^3 + 3^3 + 4^3 = $	$1 + 2 + 3 + 4 = $ .... [2]

(ii) The sum of the first 14 integers is 105.

Find the sum of the first 14 cubes.

Answer(b)(ii) .... [1]

(iii) Use the formula in **part(a)(ii)** to write down a formula for the sum of the first n cubes.

Answer(b)(iii) .... [1]

(iv) Find the sum of the first 60 cubes.

Answer(b)(iv) .... [1]

(v) Find n when the sum of the first n cubes is $278\,784$.

Answer(b)(v) $n =$ .... **[2]**

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2 **(a)** Find the integer values for x which satisfy the inequality $\qquad -3 < 2x - 1 \leqslant 6$.

Answer(a) .... **[3]**

(b) Simplify $\dfrac{x^2 + 3x - 10}{x^2 - 25}$.

Answer(b) .... **[4]**

(c) (i) Show that $\dfrac{5}{x-3} + \dfrac{2}{x+1} = 3$ can be simplified to $3x^2 - 13x - 8 = 0$.

Answer(c)(i)

[3]

(ii) Solve the equation $3x^2 - 13x - 8 = 0$.

Show all your working and give your answers correct to two decimal places.

Answer(c)(ii) x = ... or *x =* .... [4]

5 (a) Complete the table for the function $f(x) = \dfrac{x^3}{2} - 3x - 1$.

x	-3	-2	-1.5	-1	0	1	1.5	2	3	3.5
$f(x)$	-5.5		1.8	1.5		-3.5	-3.8	-3		9.9

[3]

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3.5$.

[4]
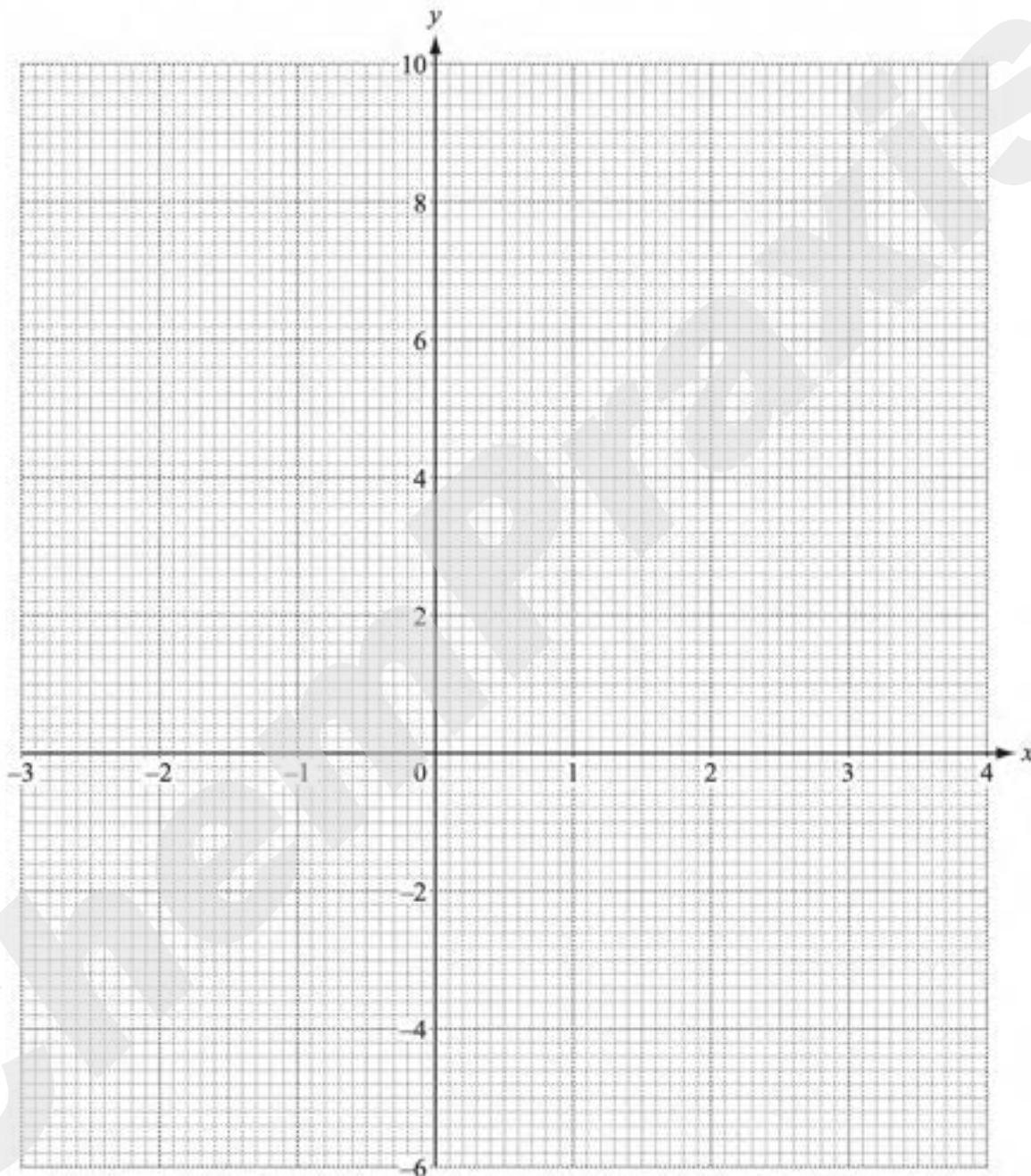
(c) Use your graph to

(i) solve f(x) = 0.5,

Answer(c)(i) x = ... or x = ... or x = ... [3]

(ii) find the inequalities for k, so that f(x) = k has only 1 answer.

Answer(c)(ii) k < ....

k > .... [2]

(d) (i) On the same grid, draw the graph of $y = 3x - 2$ for $-1 \leqslant x \leqslant 3.5$. [3]

(ii) The equation $\dfrac{x^3}{2} - 3x - 1 = 3x - 2$ can be written in the form $x^3 + ax + b = 0$.

Find the values of a and b.

Answer(d)(ii) a = ... and b = ... [2]

(iii) Use your graph to find the **positive** answers to $\dfrac{x^3}{2} - 3x - 1 = 3x - 2$ for $-3 \leqslant x \leqslant 3.5$.

Answer(d)(iii) x = ... or x = ... [2]

8   Mr Chang hires x large coaches and y small coaches to take 300 students on a school trip.
Large coaches can carry 50 students and small coaches 30 students.
There is a maximum of 5 large coaches.

(a)   Explain clearly how the following two inequalities satisfy these conditions.

     (i) $x \leqslant 5$

        Answer(a)(i)   ....   [1]

     (ii) $5x + 3y \geqslant 30$

        Answer(a)(ii)   ....

        ....   [2]

Mr Chang also knows that $x + y \leqslant 10$.

(b) On the grid, show the information above by drawing three straight lines and shading the **unwanted** regions.
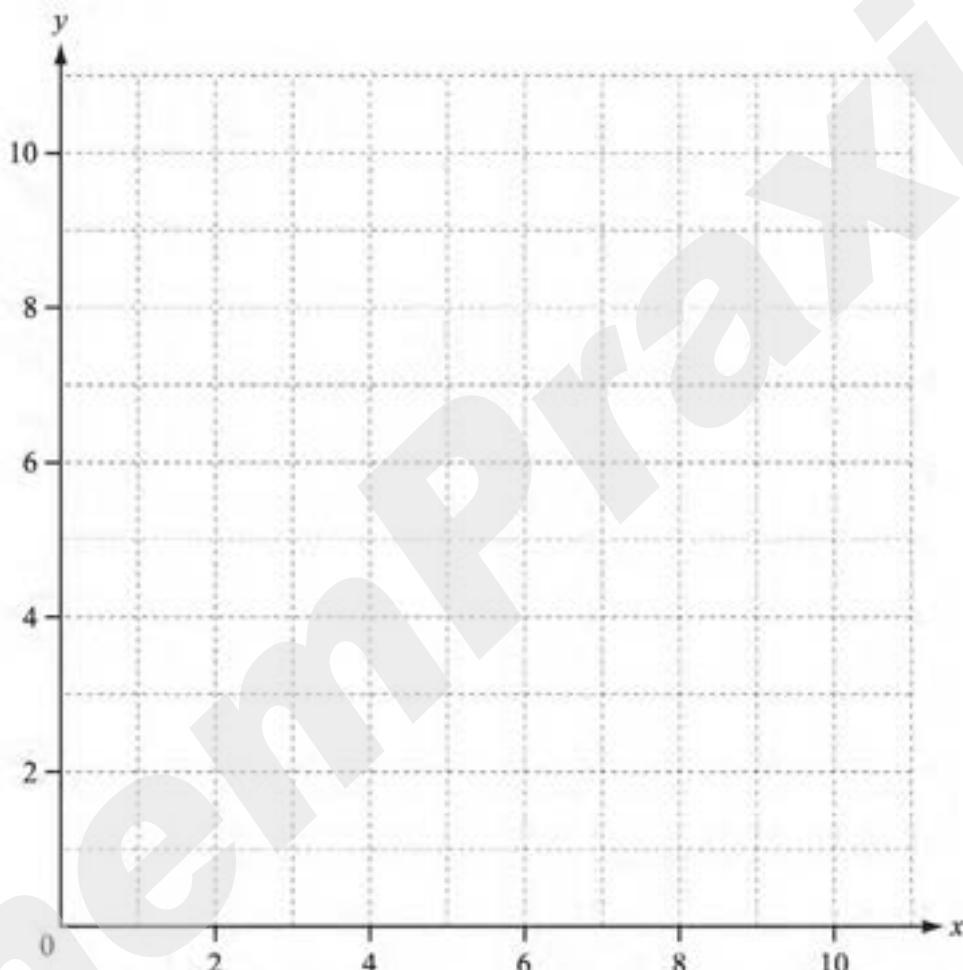
[5]

(c) A large coach costs $450 to hire and a small coach costs $350.

    (i) Find the number of large coaches and the number of small coaches that would give the minimum hire cost for this school trip.

<div align="right">

Answer(c)(i) Large coaches ....

Small coaches .... [2]
</div>

    (ii) Calculate this minimum cost.

<div align="right">

Answer(c)(ii) $ .... [1]
</div>

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Question 1b

(b) The cost to hire a tent consists of two parts.

$$\boxed{\$c} \quad + \quad \boxed{\$d \text{ per day}}$$

The total cost for 4 days is \$27.10 and for 7 days is \$34.30.

Write down two equations in c and d and solve them.

Answer(b) $c=$ ....

$d =$ .... **[4]**

2

$$f(x) = 4x - 2$$

$$g(x) = \frac{2}{x} + 1$$

$$h(x) = x^2 + 3$$

(a) (i) Find the value of hf(2).

Answer(a)(i) .... **[2]**

(ii) Write fg(x) in its simplest form.

Answer(a)(ii) fg(x) = ... [2]

(b) Solve g(x) = 0.2.

Answer(b) x = ... [2]

(c) Find the value of gg(3).

Answer(c) .... [2]

(d) (i) Show that $f(x) = g(x)$ can be written as $4x^2 - 3x - 2 = 0$.

Answer (d)(i)

[1]

(ii) Solve the equation $4x^2 - 3x - 2 = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(d)(ii) x = ... or *x =* .... [4]

7   The diagram shows the accurate graph of $y = f(x)$ where $f(x) = \dfrac{1}{x} + x^2$ for $0 < x \leqslant 3$.
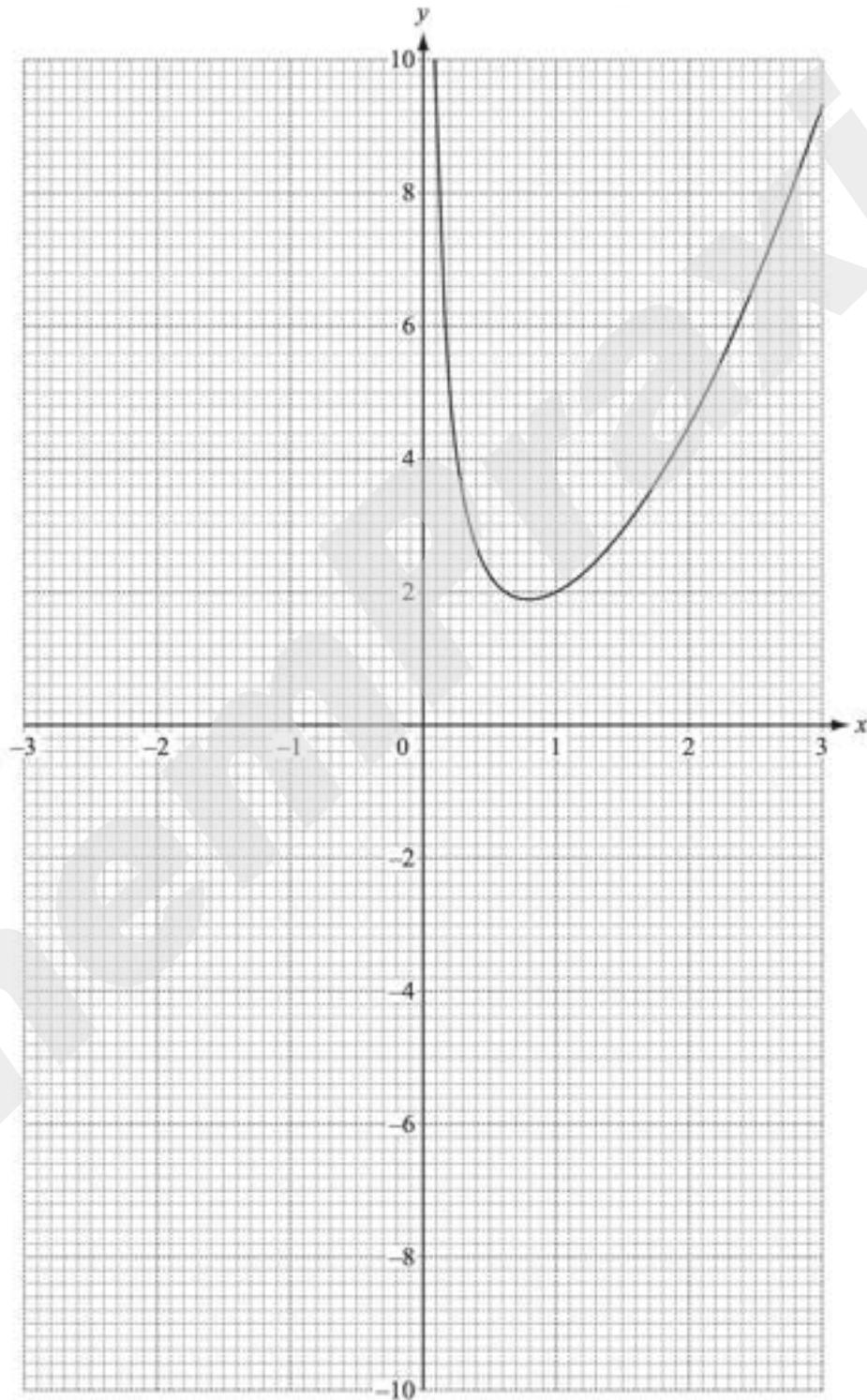
(a) Complete the table for $f(x) = \dfrac{1}{x} + x^2$.

x	-3	-2	-1	-0.5	-0.3	-0.1
$f(x)$		3.5	0	-1.8		

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leq x < 0$. [3]

(c) By drawing a tangent, work out an estimate of the gradient of the graph where $x = 2$.

Answer(c) .... [3]

(d) Write down the inequality satisfied by k when $f(x) = k$ has three answers.

Answer(d) .... [1]

(e) (i) Draw the line $y = 1 - x$ on the grid for $-3 \leq x \leq 3$. [2]

(ii) Use your graphs to solve the equation $1 - x = \dfrac{1}{x} + x^2$.

Answer(e)(ii) x = ... [1]

(f) **(i)** Rearrange $x^3 - x^2 - 2x + 1 = 0$ into the form $\dfrac{1}{x} + x^2 = ax + b$, where a and b are integers.

Answer(f)(i)

[2]

(ii) Write down the equation of the line that could be drawn on the graph to solve $x^3 - x^2 - 2x + 1 = 0$.

Answer(f)(ii) $y =$ .... [1]

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2 **(a)** Complete the table of values for $y = 2^x$.

x	-2	-1	0	1	2	3
y	0.25		1	2		8

[2]

(b) On the grid, draw the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$.

[3]
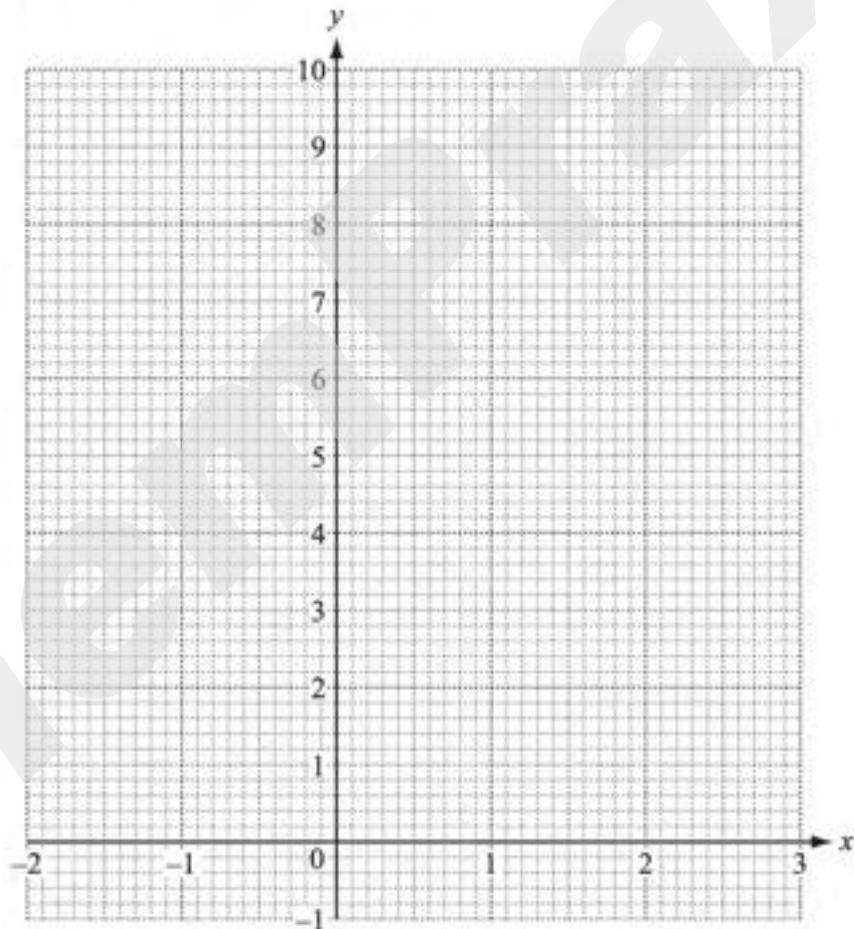
(d) Draw the tangent to the curve at the point where $x = 1$.

Use this tangent to calculate an estimate of the gradient of $y = 2^x$ when $x = 1$.

Answer(d) .... [3]

Question 5b

(b) The cost of a biscuit is x cents.
The cost of a cake is $(x + 3)$ cents.
The number of biscuits Roshni can buy for 72 cents is 2 more than the number of cakes she can buy for 72 cents.

(i) Show that $x^2 + 3x - 108 = 0$.

Answer(b)(i)

[3]

(ii) Solve the equation $x^2 + 3x - 108 = 0$.

Answer(b)(ii) x = ... or *x =* .... [3]

(iii) Find the total cost of 2 biscuits and 1 cake.

Answer(b)(iii) .... cents [1]

8 $f(x) = x^2 + x - 1$ $\qquad$ $g(x) = 1 - 2x$ $\qquad$ $h(x) = 3^x$

(a) Find the value of $hg(-2)$.

Answer(a) .... [2]

(b) Find $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ .... [2]

(c) Solve the equation $\quad f(x) = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer(c) $x =$ .... or $x =$ .... [4]

(d) Find $\quad fg(x)$.
Give your answer in its simplest form.

Answer(d) $fg(x) =$ .... [3]

(e) Solve the equation $\quad h^{-1}(x) = 2$.

Answer(e) $x =$ .... [1]

10   Hassan stores books in large boxes and small boxes.
Each large box holds 20 books and each small box holds 10 books.
He has x large boxes and y small boxes.

(a)   Hassan must store at least 200 books.

Show that $2x + y \geqslant 20$.

Answer(a)

[1]

(b)   Hassan must not use more than 15 boxes.
He must use at least 3 small boxes.
The number of small boxes must be less than or equal to the number of large boxes.

Write down three inequalities to show this information.

Answer(b)   ....

....

....   [3]

(c) On the grid, show the information in **part (a)** and **part (b)** by drawing four straight lines and shading the **unwanted** regions.
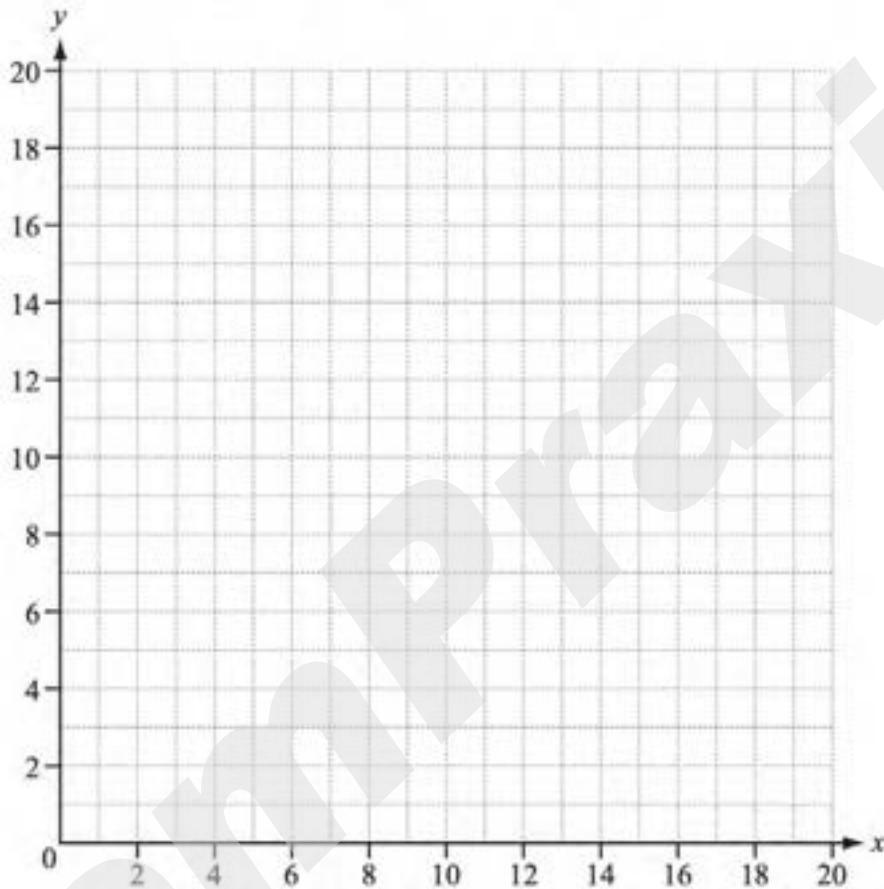
[6]

(d) A large box costs $5 and a small box costs $2.

    (i) Find the least possible total cost of the boxes.

Answer(d)(i) $ .... [1]

(ii) Find the number of large boxes and the number of small boxes which give this least possible cost.

Answer(d)(ii) Number of large boxes = ...

Number of small boxes = ... [2]

12 (a) The nth term of a sequence is $n(n+1)$.

(i) Write the two missing terms in the spaces.   2,   6, ... ,   20, ... [2]

(ii) Write down an expression in terms of n for the $(n+1)$th term.

Answer(a)(ii) .... [1]

(iii) The difference between the nth term and the $(n+1)$th term is $pn+q$.

Find the values of p and q.

Answer(a)(iii) p = ...

q = ... [2]

(iv) Find the positions of the two consecutive terms which have a difference of 140.

Answer(a)(iv) .... and .... [2]

(b) A sequence u_1, u_2, u_3, u_4, ... is given by the following rules.

$u_1 = 2$, $u_2 = 3$ and $u_n = 2u_{n-2} + u_{n-1}$ for $n \geqslant 3$.

For example, the third term is u_3 and $u_3 = 2u_1 + u_2 = 2 \times 2 + 3 = 7$.
So, the sequence is   2, 3, 7, u_4, u_5, ...

(i) Show that $u_4 = 13$.

     Answer(b)(i) .... [1]

(ii) Find the value of u_5.

                     Answer(b)(ii) $u_5 =$ .... [1]

(iii) Two consecutive terms of the sequence are 3413 and 6827.

Find the term before and the term after these two given terms.

      Answer(b)(iii) .... , 3413,   6827, ... [2]

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3 The table shows some values for the equation $y = x^3 - 2x$ for $-2 \leqslant x \leqslant 2$.

x	-2	-1.5	-1	-0.6	-0.3	0	0.3	0.6	1	1.5	2
y	-4	-0.38			0.57		-0.57			0.38	4

(a) Complete the table of values. [3]

(b) On the grid below, draw the graph of $y = x^3 - 2x$ for $-2 \leqslant x \leqslant 2$.
The first two points have been plotted for you.
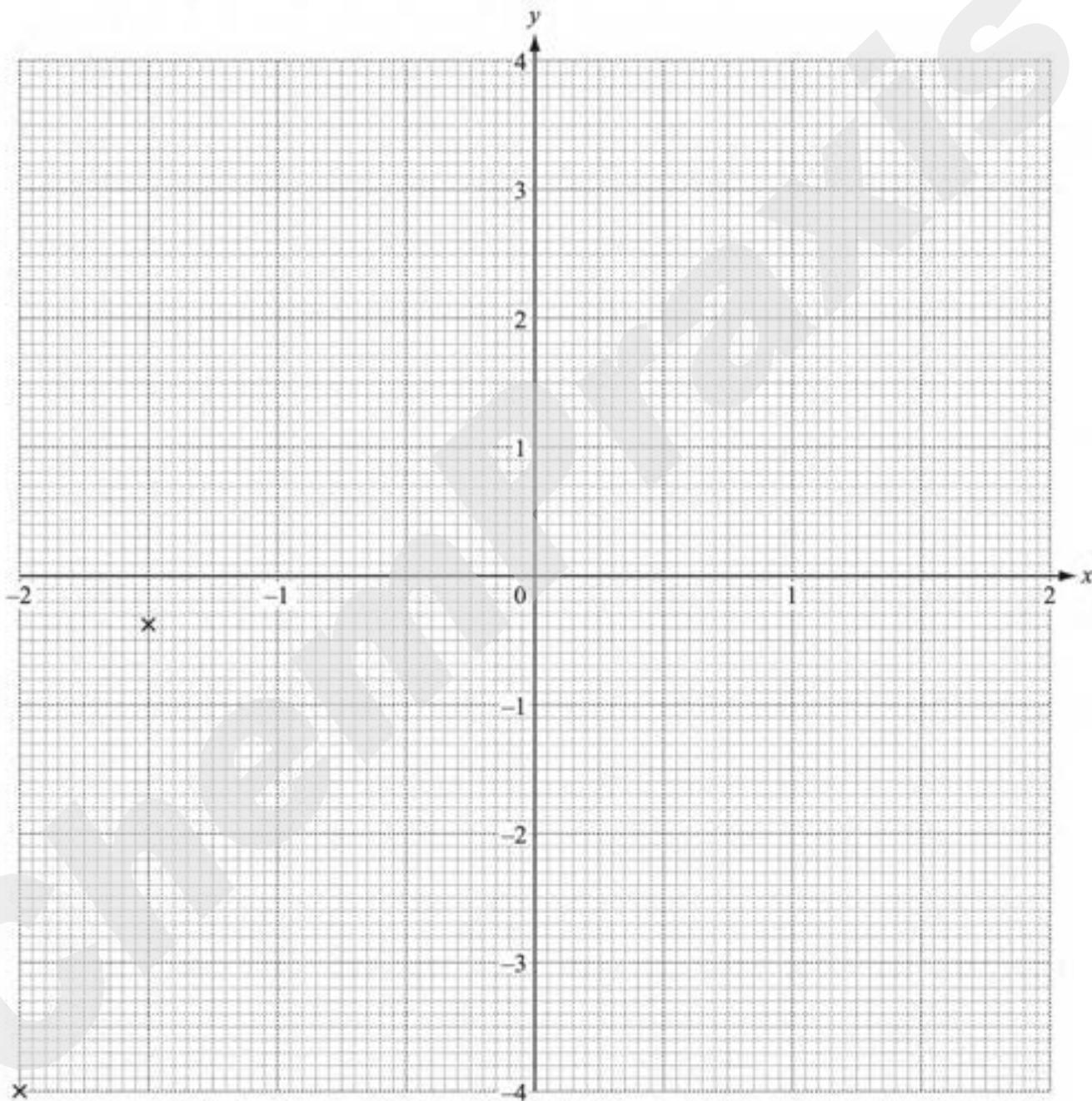
[4]

(c) **(i)** On the grid, draw the line $y = 0.8$ for $-2 \leqslant x \leqslant 2$. [1]

(ii) Use your graph to solve the equation $x^3 - 2x = 0.8$.

Answer(c)(ii) $x = $ .... or $x = $ .... or $x = $ .... [3]

(d) By drawing a suitable tangent, work out an estimate for the gradient of the graph of $y = x^3 - 2x$ where $x = -1.5$.

You must show your working.

Answer(d) .... [3]

Question 6b

(b) **(i)** Factorise $x^2 - 16$.

Answer(b)(i) .... [1]

(ii) Solve the equation $\dfrac{2x+3}{x-4} + \dfrac{x+40}{x^2-16} = 2$.

Answer(b)(ii) $x =$ .... [4]

9          $f(x) = 3x + 5$ $g(x) = 7 - 2x$ $h(x) = x^2 - 8$

(a) Find

    (i) $f(3)$,

Answer(a)(i) .... [1]

    (ii) $g(x - 3)$ in terms of x in its simplest form,

Answer(a)(ii) .... [2]

    (iii) $h(5x)$ in terms of x in its simplest form.

Answer(a)(iii) .... [1]

(b) Find the inverse function $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ .... [2]

(c) Find hf(x) in the form $ax^2 + bx + c$.

Answer(c) hf$(x) =$ .... [3]

(d) Solve the equation ff$(x) = 83$.

Answer(d) $x =$ .... [3]

(e) Solve the inequality $2f(x) < g(x)$.

Answer(e) .... [3]

3   Pablo plants x lemon trees and y orange trees.

(a) (i)   He plants at least 4 lemon trees.

Write down an inequality in x to show this information.

Answer(a)(i) .... [1]

(ii) Pablo plants at least 9 orange trees.

Write down an inequality in y to show this information.

Answer(a)(ii) .... [1]

(iii) The greatest possible number of trees he can plant is 20.

Write down an inequality in x and y to show this information.

Answer(a)(iii) .... [1]

(b) Lemon trees cost \$5 each and orange trees cost \$10 each.

The maximum Pablo can spend is \$170.

Write down an inequality in x and y and show that it simplifies to $x + 2y \leqslant 34$.

Answer (b)

[1]

(c) (i)   On the grid opposite, draw four lines to show the four inequalities and shade the **unwanted** region.
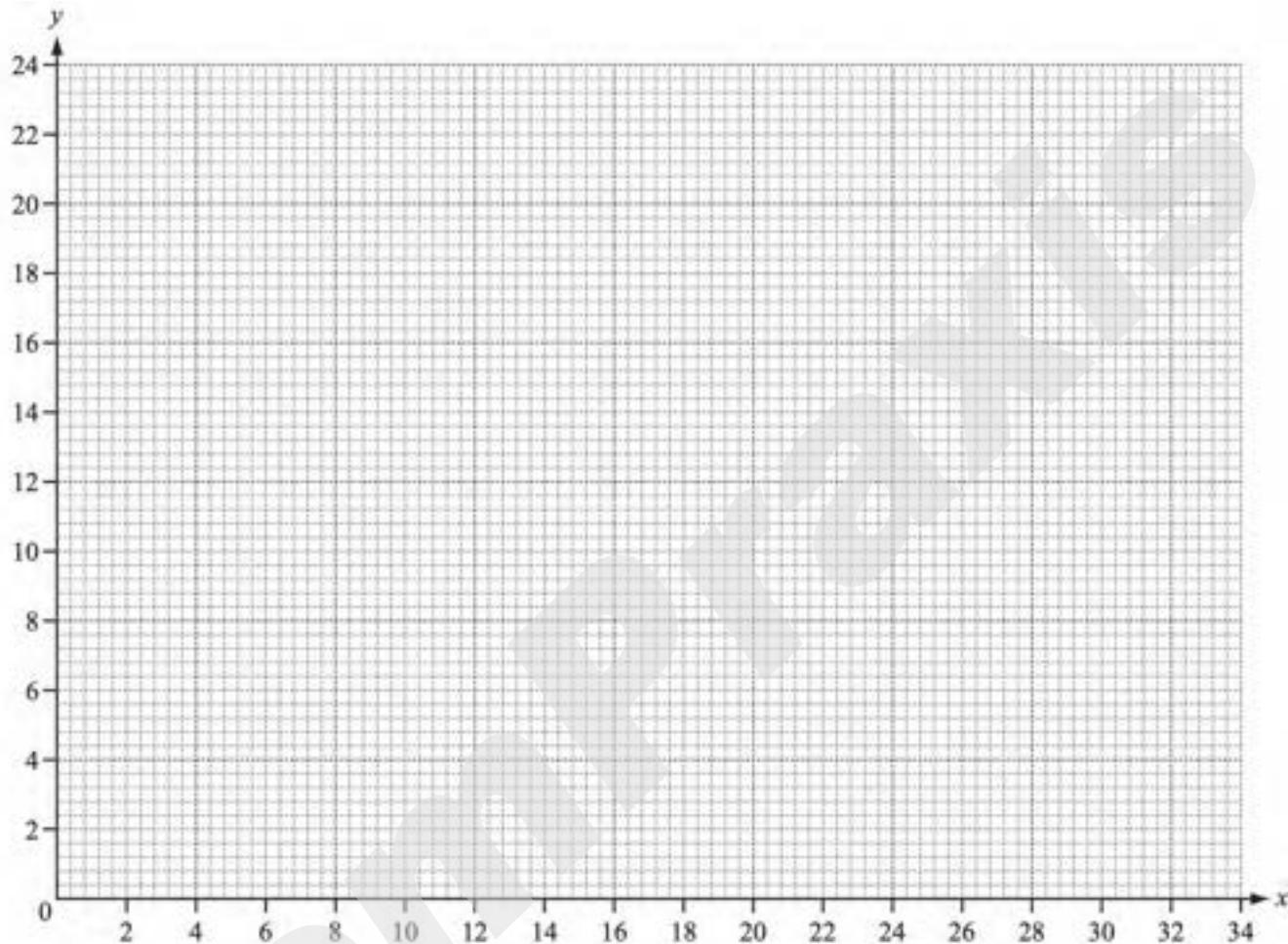
[7]

(ii) Calculate the smallest cost when Pablo buys a total of 20 trees.

Answer(c)(ii) $ .... [2]

84

Question 6b

(b) **(i)** Write the four missing terms in the table for sequences A, B, C and D.

Term	1	2	3	4	5		n
Sequence A	−4		2	5	8		$3n - 7$
Sequence B	1	4	9	16	25		
Sequence C	5	10	15	20	25		
Sequence D	6	14	24	36	50		

[4]

(ii) Which term in sequence D is equal to 500?

Answer(b)(ii) .... [2]

(c) Simplify $\dfrac{x^2 - 16}{2x^2 + 7x - 4}$.

Answer(c) .... [4]

10 (a) Simplify

(i) $(2x^2y^3)^3$,

Answer(a)(i) .... [2]

(ii) $\left(\dfrac{27}{x^6}\right)^{-\frac{1}{3}}$.

Answer(a)(ii) .... [3]

(b) Multiply out and simplify.

$$(3x - 2y)(2x + 5y)$$

Answer(b) .... [3]

(c) Make h the subject of

(i) $V = \pi r^3 + 2\pi r^2 h$,

Answer(c)(i) $h =$ .... [2]

(ii) $V = \sqrt{3h}$.

Answer(c)(ii) $h =$ .... [2]

(d) Write as a single fraction in its simplest form.

$$\frac{x}{2} + \frac{5x}{3} - \frac{7x}{4}$$

Answer(d) .... [2]

12 (a) The cost of 1 kg of tomatoes is $x and the cost of 1 kg of onions is $y.

Ian pays a total of $10.70 for 10 kg of tomatoes and 4 kg of onions.

Jao pays a total of $10.10 for 8 kg of tomatoes and 6 kg of onions.

Write down simultaneous equations and solve them to find x and y.

Answer(a) x = ...

y = ... [6]

(b) Solve $2x^2 - 5x - 8 = 0$.

Give your answers correct to 2 decimal places.
Show all your working.

Answer(b) x = ... or *x =* .... [4]

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7         $f(x) = 2^x$

(a) Complete the table.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$		1.4	2	2.8	4	5.7	8		

[3]

(b) Draw the graph of $y = f(x)$ for $0 \leqslant x \leqslant 4$.

[4]
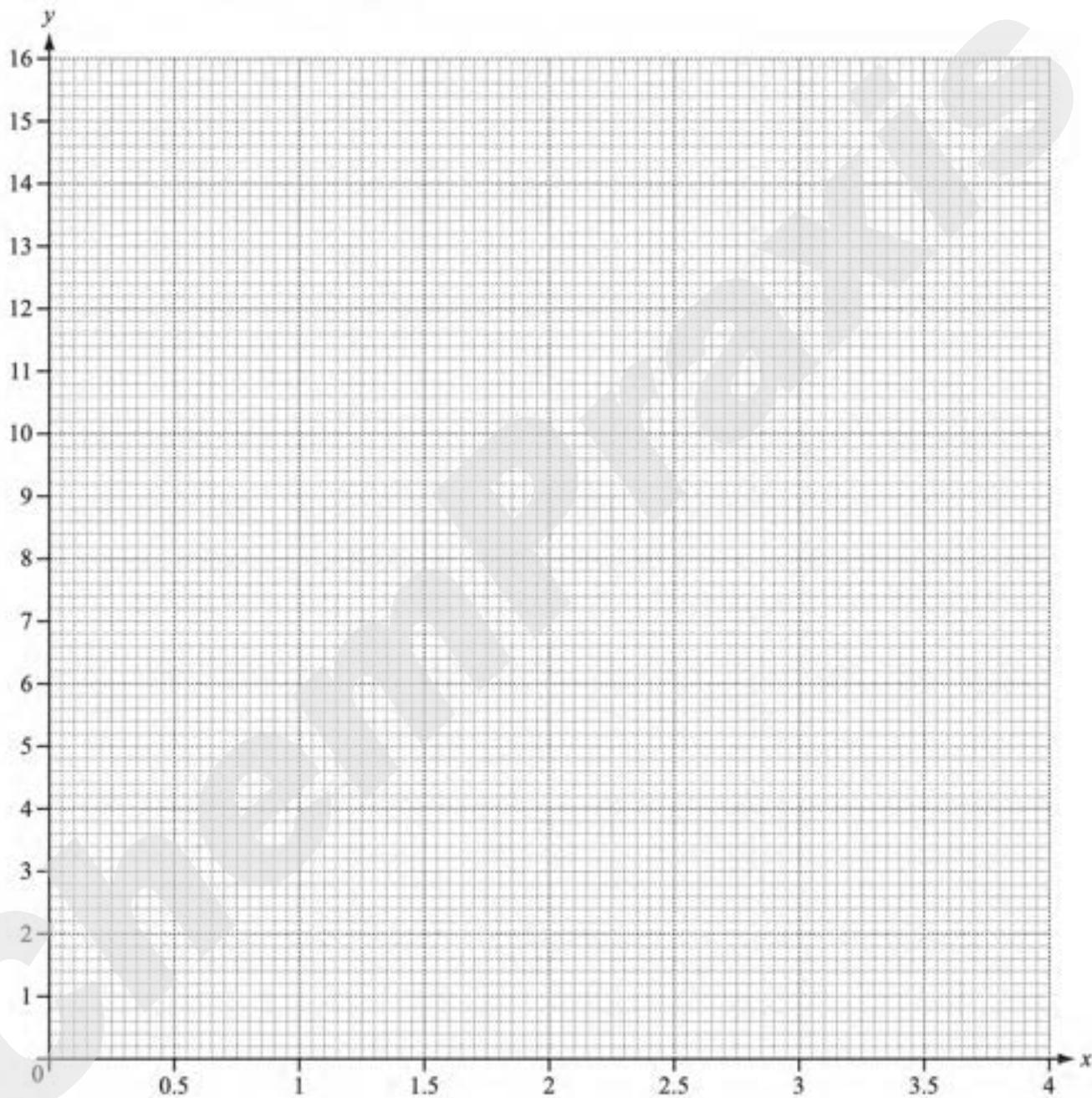
(c) Use your graph to solve the equation $2^x = 5$.

Answer(c) x = ... [1]

(d) Draw a suitable straight line and use it to solve the equation $2^x = 3x$.

Answer(d) x = ... *or x =* .... [3]

(e) Draw a suitable tangent and use it to find the co-ordinates of the point on the graph of $y = f(x)$ where the gradient of the graph is 3.

Answer(e) (............. *,* .... *)* [3]

9        $f(x) = 1 - 2x$ $g(x) = \dfrac{1}{x}, \; x \neq 0$ $h(x) = x^3 + 1$

(a) Find the value of

    (i) gf(2),

Answer(a)(i) .... [2]

    (ii) h(−2).

Answer(a)(ii) .... [1]

(b) Find fg(x).
Write your answer as a single fraction.

Answer(b) fg(x) = ... [2]

(c) Find h^{-1}(x) , the inverse of h(x).

Answer(c) h^{-1}(x) = ... [2]

(d) Write down which of these sketches shows the graph of each of $y = f(x)$, $y = g(x)$ and $y = h(x)$.

Graph A

Graph B

Graph C

Graph D
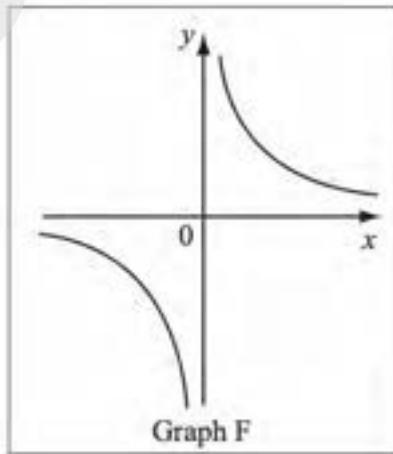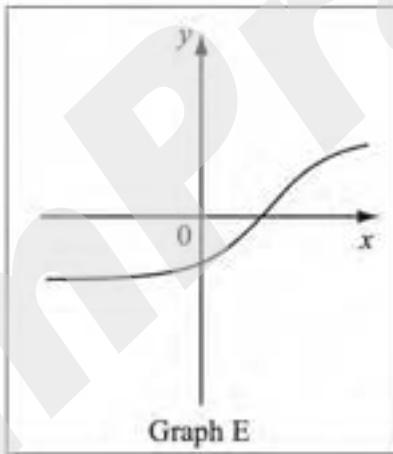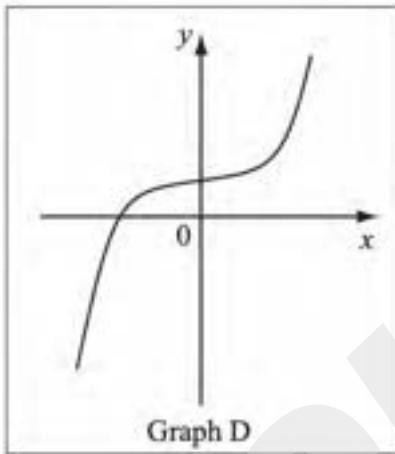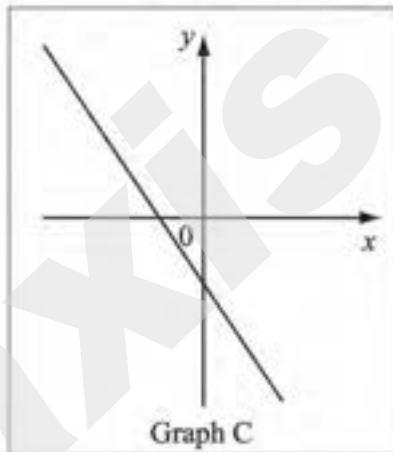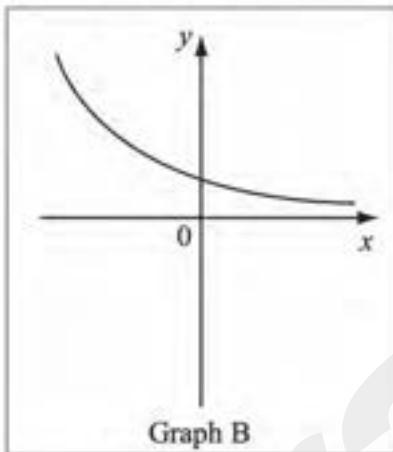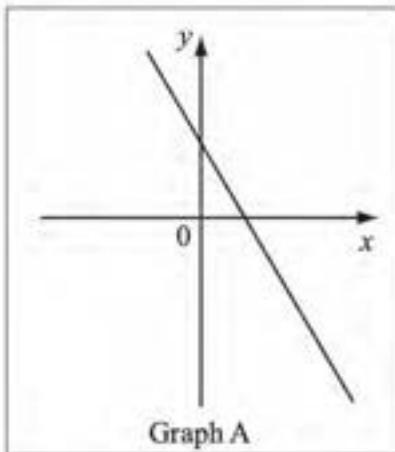
Graph E

Graph F

Answer(d) $y = f(x)$ Graph ....

$y = g(x)$ Graph ....

$y = h(x)$ Graph .... [3]

(e) $k(x) = x^5 - 3$

Solve the equation $k^{-1}(x) = 2$.

Answer(e) x = ... [2]

10 (a) Rice costs \$x per kilogram.
Potatoes cost \$(x + 1) per kilogram.
The total cost of 12 kg of rice and 7 kg of potatoes is \$31.70.

Find the cost of 1 kg of rice.

Answer(a) \$ .... [3]

(b) The cost of a small bottle of juice is \$y.
The cost of a large bottle of juice is \$(y + 1).
When Catriona spends \$36 on small bottles only, she receives 25 more bottles than when she spends \$36 on large bottles only.

(i) Show that $25y^2 + 25y - 36 = 0$.

Answer(b)(i)

[3]

(ii) Factorise $25y^2 + 25y - 36$.

Answer(b)(ii) .... [2]

(iii) Solve the equation $25y^2 + 25y - 36 = 0$.

Answer(b)(iii) y = ... or *y =* .... [1]

(iv) Find the total cost of 1 small bottle of juice and 1 large bottle of juice.

*Answer(b)(iv)* \$ .... [1]

11
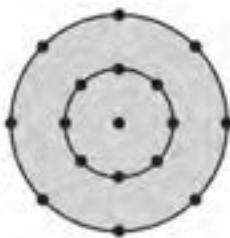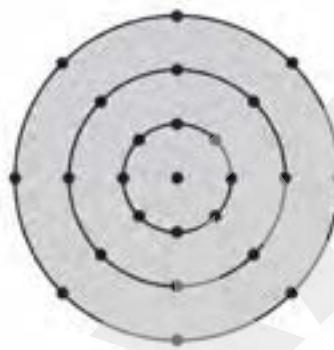
| Diagram 1 | Diagram 2 | Diagram 3 |

The diagrams show a sequence of dots and circles.
Each diagram has one dot at the centre and 8 dots on each circle.
The radius of the first circle is 1 unit.
The radius of each new circle is 1 unit greater than the radius of the previous circle.

(a) Complete the table for diagrams 4 and 5.

Diagram	1	2	3	4	5
Number of dots	9	17	25		
Area of the largest circle	π	4π	9π		
Total length of the circumferences of the circles	2π	6π	12π		

[4]

(b) **(i)** Write down, in terms of n, the number of dots in diagram n.

Answer(b)(i) .... [2]

(ii) Find n, when the number of dots in diagram n is 1097.

Answer(b)(ii) n = ... [2]

(c) Write down, in terms of n and π, the area of the largest circle in

    (i) diagram n,

Answer(c)(i) .... [1]

    (ii) diagram $3n$.

Answer(c)(ii) .... [1]

(d) Find, in terms of n and π, the total length of the circumferences of the circles in diagram n.

Answer(d) .... [2]

October/November 2012 (41)

2   **(a)** **(i)** Complete the table of values for $y = \frac{1}{2}x^3 + x^2 - 7x.$

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	-2.5	12	16.5		7.5	0		-6	1.5	

[3]

(ii) On the grid, draw the graph of $y = \frac{1}{2}x^3 + x^2 - 7x$ for $-5 \leqslant x \leqslant 4$.
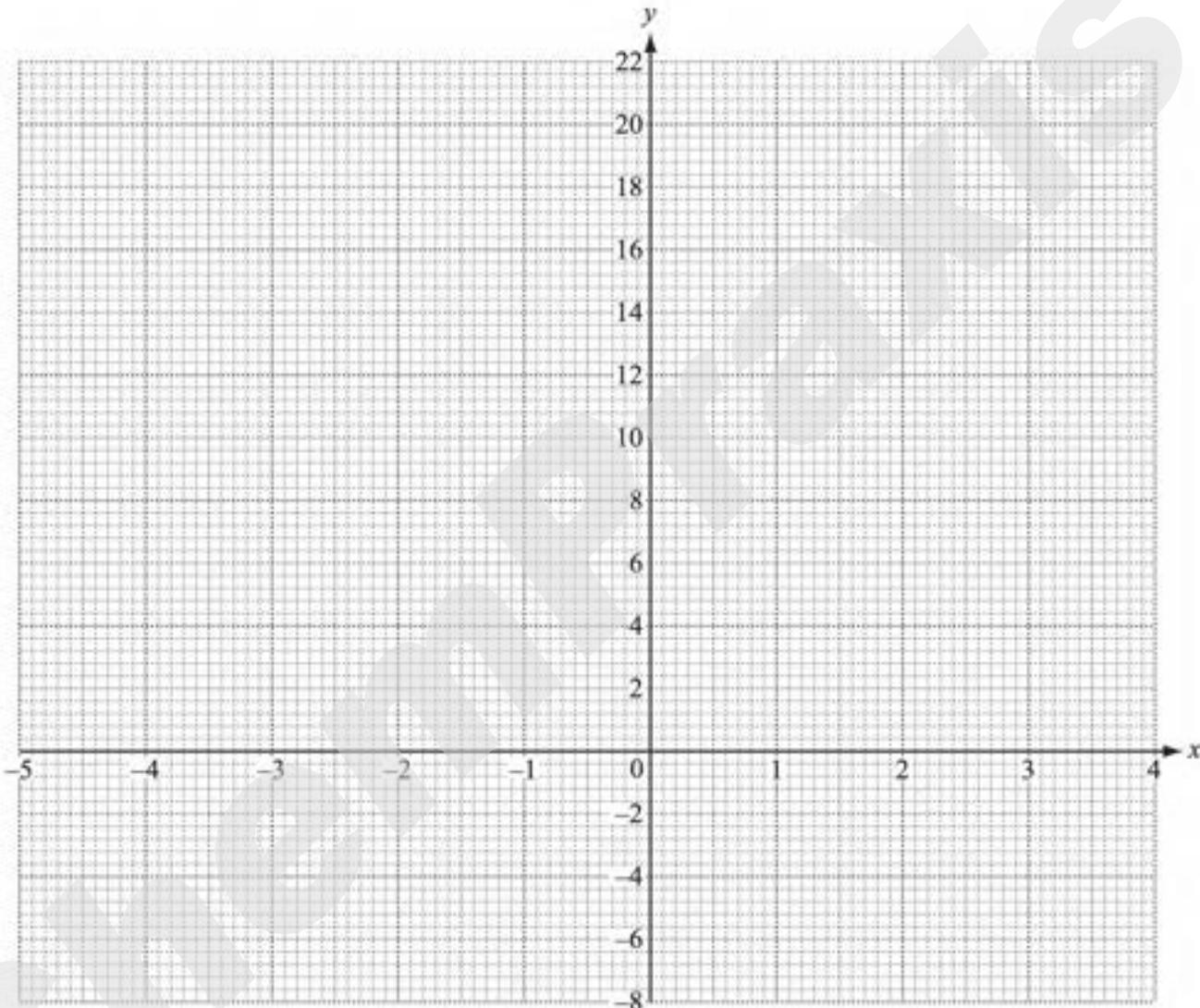
[4]

(b) Use your graph to solve the equation $\frac{1}{2}x^3 + x^2 - 7x = 2$.

Answer(b) $x =$ .... or $x =$ .... or $x =$ .... [3]

(c) By drawing a suitable tangent, calculate an estimate of the gradient of the graph where $x = -4$.

Answer(c) .... [3]

(d) **(i)** On the grid draw the line $y = 10 - 5x$ for $-2 \leqslant x \leqslant 3$. [3]

(ii) Use your graphs to solve the equation $\frac{1}{2}x^3 + x^2 - 7x = 10 - 5x$.

Answer(d)(ii) $x =$ .... [1]

4 **(a)** Solve the equations.

(i) $4x - 7 = 8 - 2x$

Answer(a)(i) $x =$ .... [2]

(ii) $\dfrac{x-7}{3} = 2$

Answer(a)(ii) $x =$ .... [2]

(b) Simplify the expressions.

(i) $(3xy^4)^3$

Answer(b)(i) .... [2]

(ii) $(16a^6b^2)^{\frac{1}{2}}$

Answer(b)(ii) .... [2]

(iii) $\dfrac{x^2 - 7x - 8}{x^2 - 64}$

Answer(b)(iii) .... [4]

3 **(a)** **(i)** Factorise completely the expression $4x^2 - 18x - 10$.

Answer(a)(i) .... [3]

(ii) Solve $4x^2 - 18x - 10 = 0$.

Answer(a)(ii) $x =$ .... or $x =$ .... [1]

(b) Solve the equation $2x^2 - 7x - 10 = 0$.

Show all your working and give your answers correct to two decimal places.

Answer(b) $x =$ .... or $x =$ .... [4]

(c) Write $\dfrac{6}{3x-1} - \dfrac{2}{x-2}$ as a single fraction in its simplest form.

Answer(c) .... [3]

7 Jay makes wooden boxes in two sizes. He makes x small boxes and y large boxes.
He makes at least 5 **small** boxes.
The greatest number of **large** boxes he can make is 8.
The greatest total number of boxes is 14.
The number of **large** boxes is at least half the number of **small** boxes.

(a) (i) Write down four inequalities in x and y to show this information.

Answer(a)(i) ....

....

....

.... [4]

(ii) Draw four lines on the grid and write the letter R in the region which represents these inequalities.

[5]
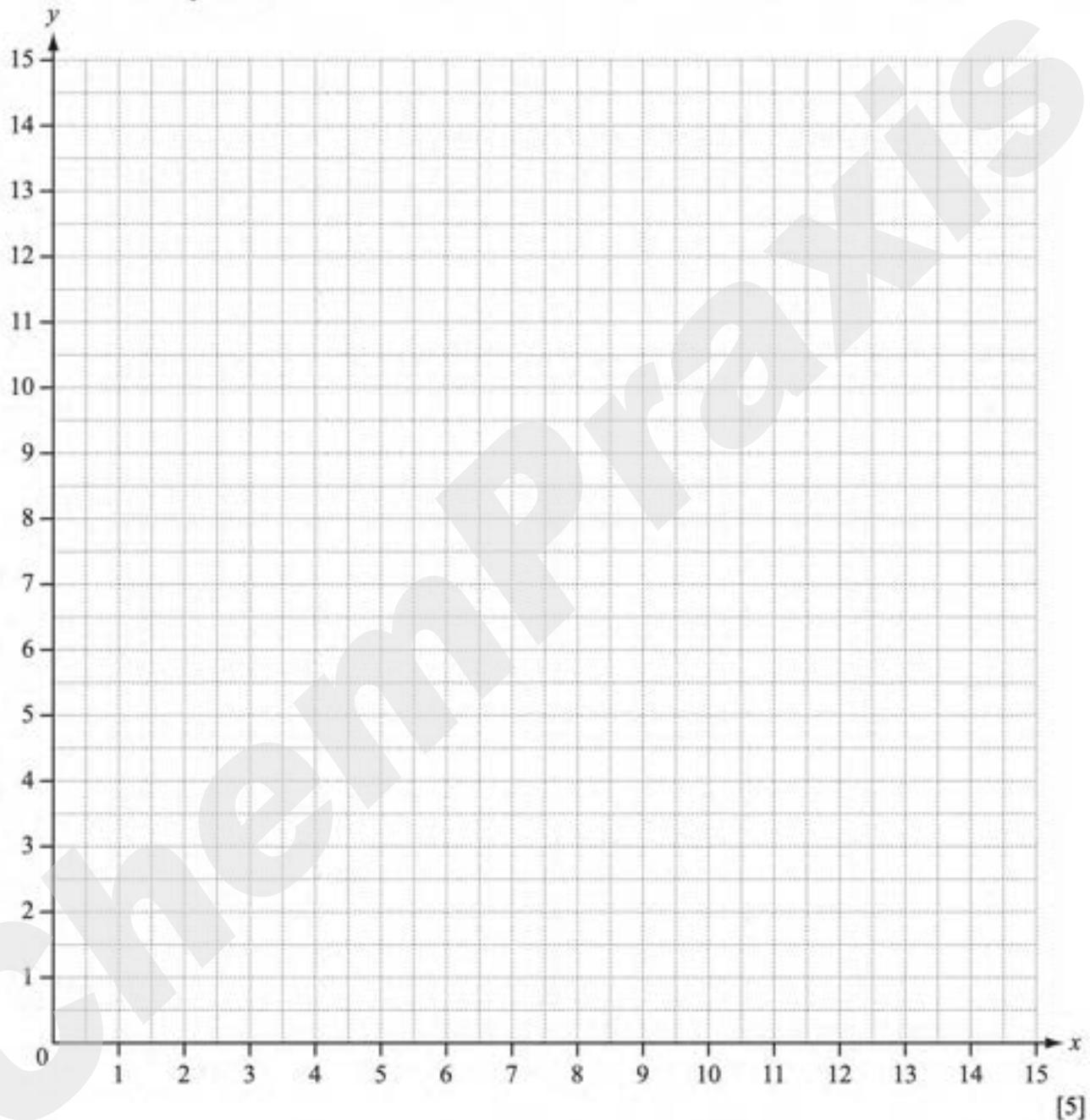
(b) The price of the small box is $20 and the price of the large box is $45.

(i) What is the greatest amount of money he receives when he sells all the boxes he has made?

Answer(b)(i) $ .... [2]

(ii) For this amount of money, how many boxes of each size did he make?

Answer(b)(ii) .... small boxes and .... large boxes [1]

8 The graph of $y = f(x)$ is drawn on the grid for $0 \leqslant x \leqslant 3.2$.
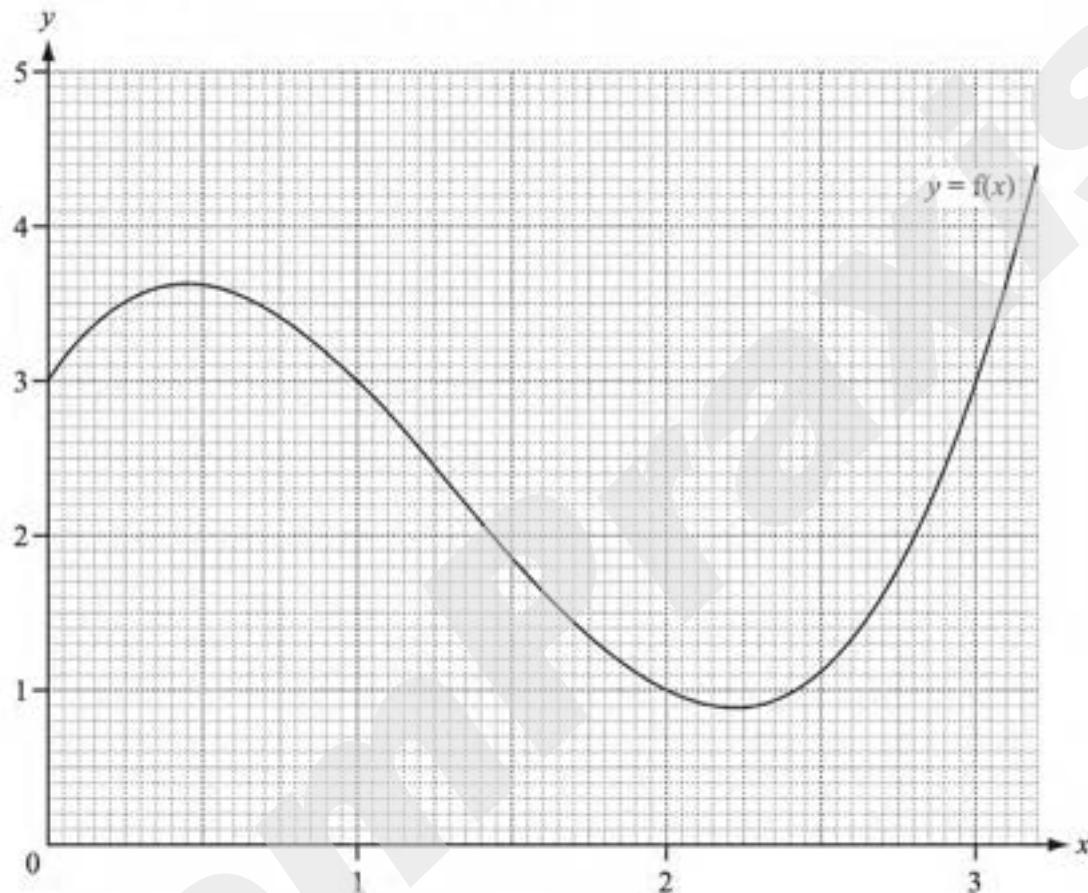
(a) (i) Draw the tangent to the curve $y = f(x)$ at $x = 2.5$. [1]

(ii) Use your tangent to estimate the gradient of the curve at $x = 2.5$.

Answer(a)(ii) .... [2]

(b) Use the graph to solve $f(x) = 2$, for $0 \leqslant x \leqslant 3.2$.

Answer(b) $x =$ .... or $x =$ .... [2]

(c) $$g(x) = \frac{x}{2} + \frac{2}{x^2} \quad x \neq 0.$$

(i) Complete the table for values of $g(x)$, correct to 1 decimal place.

x	0.7	1	1.5	2	2.5	3
$g(x)$			1.6		1.6	1.7

[2]

(ii) On the grid opposite, draw the graph of $y = g(x)$ for $0.7 \leqslant x \leqslant 3$. [3]

(iii) Solve $f(x) = g(x)$ for $0.7 \leqslant x \leqslant 3$.

Answer(c) (iii) $x =$ .... or $x =$ .... or $x =$ .... [3]

10 Consecutive integers are set out in rows in a grid.

(a) This grid has 5 columns.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
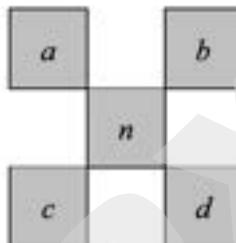
The shape drawn encloses five numbers 7, 9, 13, 17 and 19. This is the $n = 13$ shape.

In this shape, $a = 7$, $b = 9$, $c = 17$ and $d = 19$.

(i) Calculate $bc - ad$ for the $n = 13$ shape.

Answer(a)(i) .... [1]

(ii) For the 5 column grid, $a = n - 6$.

Write down b, c and d in terms of n for this grid.

Answer(a)(ii) $b =$ ....

$c =$ ....

$d =$ .... [2]

(iii) Write down $bc - ad$ in terms of n.
Show clearly that it simplifies to 20.

Answer(a)(iii)

[2]

(b) This grid has 6 columns. The shape is drawn for $n = 10$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

a		b
	n	
c		d
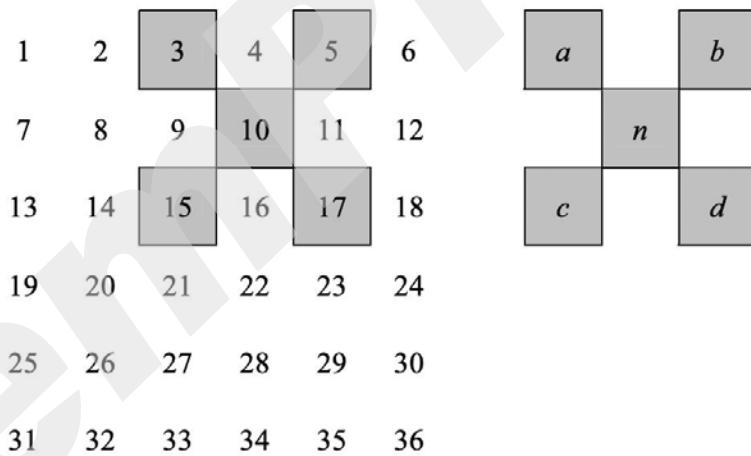
(i) Calculate the value of $bc - ad$ for $n = 10$.

Answer(b)(i) .... [1]

(ii) Without simplifying, write down $bc - ad$ in terms of n for this grid.

Answer(b)(ii) .... [2]

(c) This grid has 7 columns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

a		b
	n	
c		d
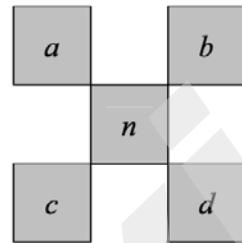
Show clearly that $bc - ad = 28$ for $n = 17$.

Answer(c)

[1]

(d) Write down the value of $bc - ad$ when there are t columns in the grid.

Answer(d) .... [1]

(e) Find the values of c, d and $bc - ad$ for this shape.

2	3	4
	16	
c		d
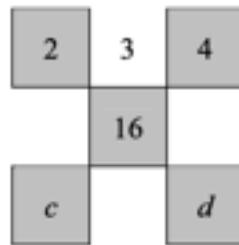
Answer (e) c = ...

$d =$ ....

$bc - ad =$ .... **[2]**

October/November 2012 (43)

4

$$f(x) = \frac{2}{x^2} - 3x, \; x \neq 0$$

(a) Complete the table.

x	−3	−2.5	−2	−1.5	−1	−0.5	0.5	1	1.5	2	2.5	3
$f(x)$	9.2	7.8	6.5	5.4		9.5	6.5		−3.6	−5.5	−7.2	−8.8

[2]

(b) On the grid, draw the graph of $y = f(x)$, for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$.

[5]
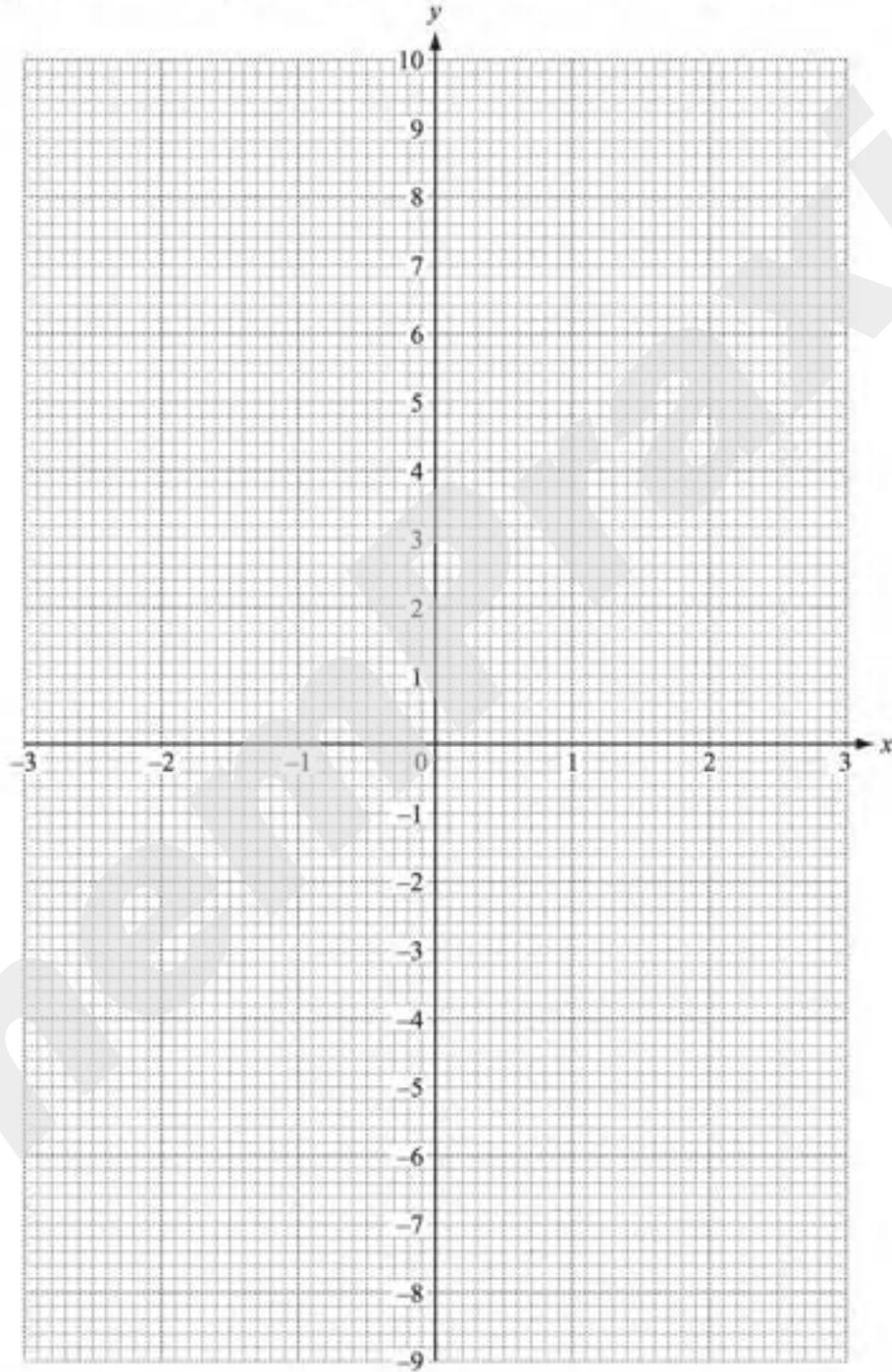
(c) Use your graph to solve the equations.

    (i) $f(x) = 4$

Answer(c)(i) $x =$ .... [1]

    (ii) $f(x) = 3x$

Answer(c)(ii) $x =$ .... [2]

(d) The equation $f(x) = 3x$ can be written as $x^3 = k$.

    Find the value of k.

Answer(d) $k =$ .... [2]

(e) (i) Draw the straight line through the points $(-1, 5)$ and $(3, -9)$.   [1]

    (ii) Find the equation of this line.

*Answer(e)*(ii) .... [3]

(iii) Complete the statement.

The straight line in **part (e)(ii)** is a .... to the graph of $y = f(x)$. [1]

5   (a) Marcos buys 2 bottles of water and 3 bottles of lemonade.
The total cost is $3.60.
The cost of one bottle of lemonade is $0.25 more than the cost of one bottle of water.
Find the cost of one bottle of water.

Answer(a) $ .... [4]

(b)
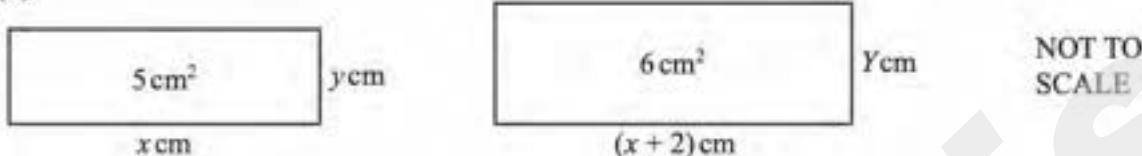
The diagram shows two rectangles.
The first rectangle measures x cm by y cm and has an area of $5\,cm^2$.
The second rectangle measures $(x + 2)$ cm by Y cm and has an area of $6\,cm^2$.

(i) When $y + Y = 1$, show that $x^2 - 9x - 10 = 0$.

Answer (b)(i)

[4]

(ii) Factorise $x^2 - 9x - 10$.

Answer(b)(ii) .... [2]

(iii) Calculate the perimeter of the first rectangle.

Answer(b)(iii) .... cm [2]

(c)
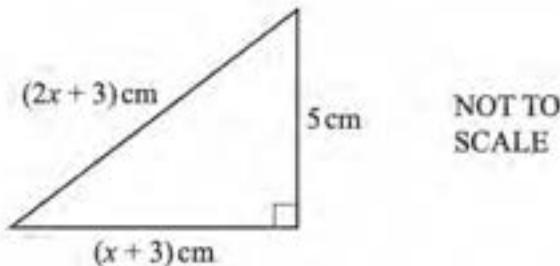
The diagram shows a right-angled triangle with sides of length $5\,cm$, $(x+3)\,cm$ and $(2x+3)\,cm$.

(i) Show that $3x^2 + 6x - 25 = 0$.

Answer (c)(i)

[4]

(ii) Solve the equation $3x^2 + 6x - 25 = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer(c)(ii) x = ... *or x =* .... [4]

10 (a) Complete the table for the 6th term and the nth term in each sequence.

	Sequence	6th term		nth term
A	11, 9, 7, 5, 3			
B	1, 4, 9, 16, 25			
C	2, 6, 12, 20, 30			
D	3, 9, 27, 81, 243			
E	1, 3, 15, 61, 213			

[12]

(b) Find the value of the 100th term in

(i) Sequence A,

Answer(b)(i) .... [1]

(ii) Sequence C.

Answer(b)(ii) .... [1]

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(c) Find the value of n in Sequence D when the nth term is equal to 6561.

Answer(c) n = ... [1]

(d) Find the value of the 10th term in Sequence E.

Answer(d) .... [1]

May/June 2013 (41)

2

$$f(x) = 3 - x - x^2 \qquad g(x) = 3^x$$

(a) Complete the tables of values for $f(x)$ and $g(x)$.

x	−1.5	−1	−0.5	0	0.5	1	1.5
f(x)	2.25	3	3.25		2.25	1	−0.75

x	−1.5	−1	−0.5	0	0.5	1	1.5
g(x)	0.19		0.58		1.73	3	5.20

[3]

(b) On the grid, draw the graphs of $y = f(x)$ and $y = g(x)$ for $-1.5 \leqslant x \leqslant 1.5$.
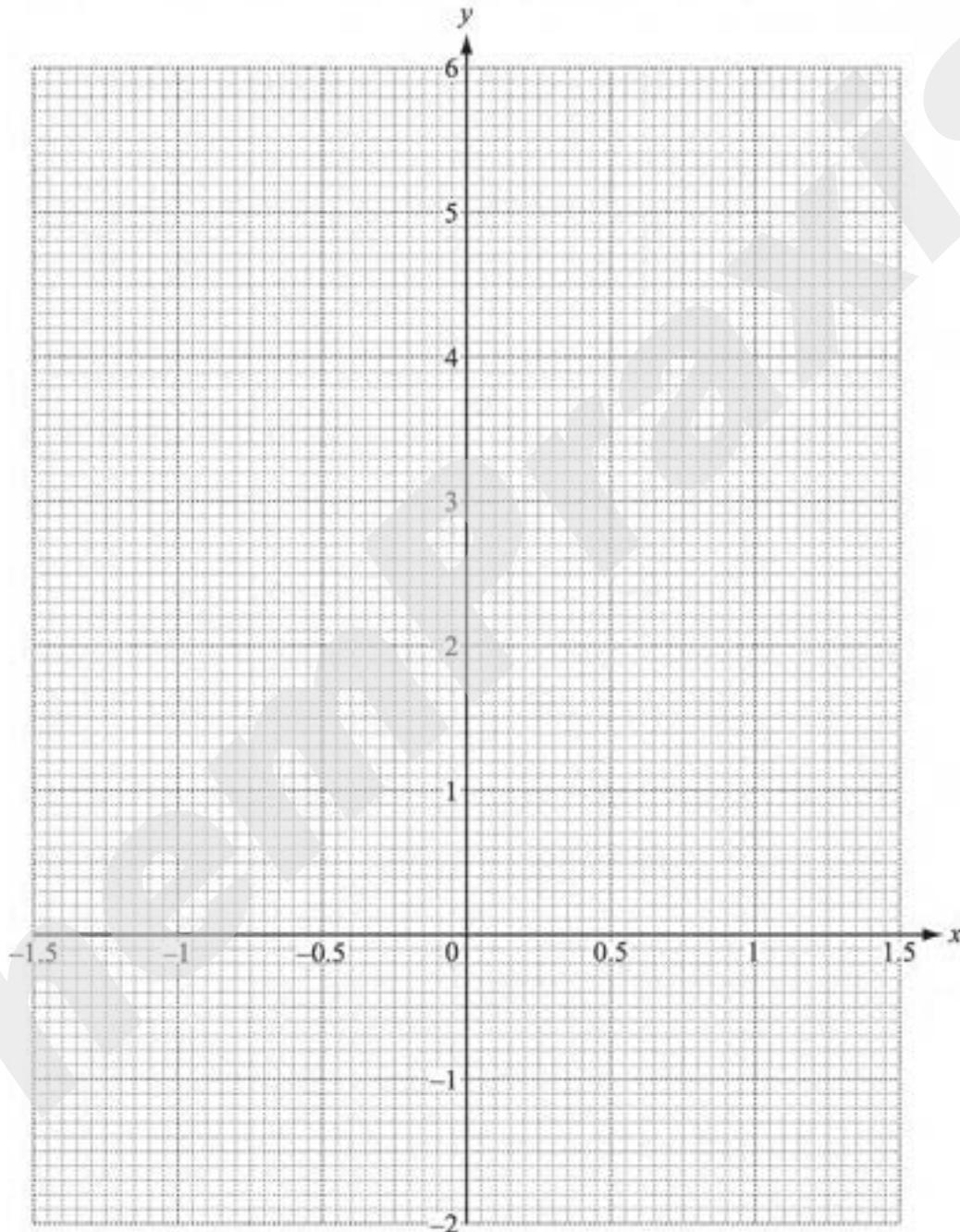
[6]

(c) For $-1.5 \leqslant x \leqslant 1.5$, use your graphs to solve

(i) $f(x) = 0$,

Answer(c)(i) $x =$ .... [1]

(ii) $g(x) = 4$,

Answer(c)(ii) $x =$ .... [1]

(iii) $f(x) = g(x)$.

Answer(c)(iii) $x =$ .... [1]

(d) By drawing a suitable tangent, find an estimate of the gradient of the graph of $y = f(x)$ when $x = 0.5$.

Answer(d) .... [3]

9     $f(x) = x^2 + x - 3$ $g(x) = 2x + 7$ $h(x) = 2^x$

(a) Solve the equation $f(x) = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer(a) $x =$ .... or $x =$ .... [4]

(b) $fg(x) = px^2 + qx + r$

Find the values of p, q and r.

Answer(b) $p =$ ....

$q =$ ....

$r =$ .... [3]

(c) Find $g^{-1}(x)$.

Answer(c) $g^{-1}(x) =$ .... [2]

(d) Find x when $h(x) = 0.25$.

Answer(d) $x =$ .... [1]

(e) Find $hhh(3)$.
Give your answer in standard form, correct to 4 significant figures.

Answer(e) .... [4]

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10
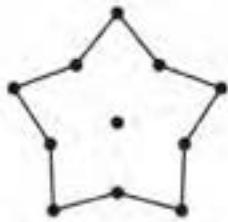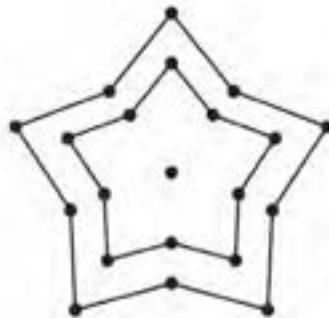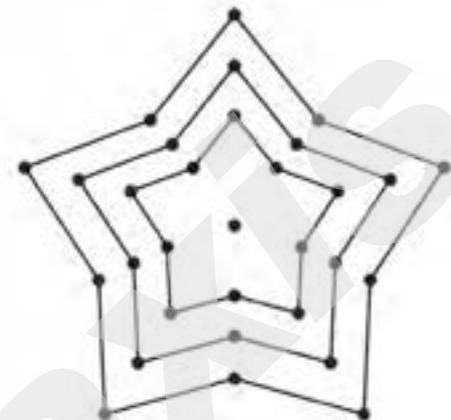
Star 1   Star 2   Star 3

The diagrams show a sequence of stars made of lines and dots.

(a) Complete the table for Star 5, Star 7 and Star n.

	Star 1	Star 2	Star 3	Star 4	Star 5		Star 7		Star n
Number of lines	10	20	30	40					
Number of dots	11	21	31	41					

[4]

(b) The sums of the number of dots in two consecutive stars are shown in the table.

Star 1 and Star 2	Star 2 and Star 3	Star 3 and Star 4
32	52	72

Find the sum of the number of dots in

(i) Star 10 and Star 11,

Answer (b)(i) .... [1]

(ii) Star n and Star $(n + 1)$,

Answer(b)(ii) .... [1]

(iii) Star $(n + 7)$ and Star $(n + 8)$.

Answer(b)(iii) .... [1]

(c) The **total number of dots** in the first n stars is given by the expression $5n^2 + 6n$.

  (i) Show that this expression is correct when $n = 3$.

  Answer(c)(i)

  [2]

  (ii) Find the total number of dots in the first 10 stars.

  Answer(c)(ii) .... [1]

(d) The total number of dots in the first n stars is $5n^2 + 6n$.
The number of dots in the $(n + 1)$th star is $10(n + 1) + 1$.

Add these two expressions to show that the total number of dots in the first $(n + 1)$ stars is

$$5(n + 1)^2 + 6(n + 1).$$

You must show each step of your working.

Answer(d)

[4]

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3   The table shows some values for the function $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.

x	1	1.5	2	2.5	3	3.5	4	4.5
y	−3		2	3	3			

(a)   Complete the table of values.   [3]

(b)   On the grid below, draw the graph of $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.

[4]
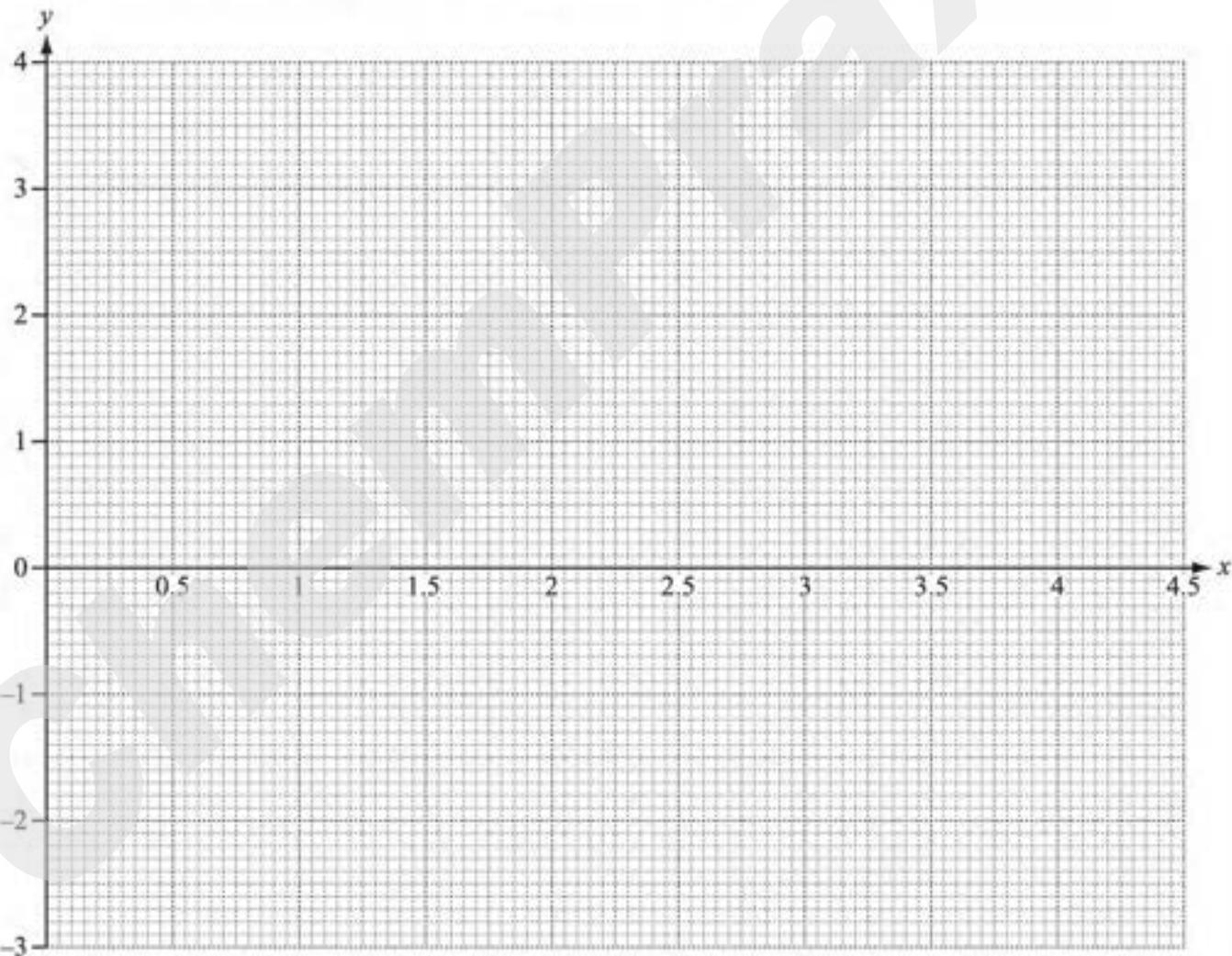
(c) By drawing a suitable line, use your graph to solve the equation $11x - 2x^2 = 11$.

Answer(c) $x =$ .... or $x =$ .... [2]

5 Paul buys a number of large sacks of fertiliser costing $\$x$ each.

He spends $27.

(a) Write down, in terms of x, an expression for the number of large sacks which Paul buys.

Answer(a) .... [1]

(b) Rula buys a number of small sacks of fertiliser.
Each small sack costs $2 less than a large sack.
Rula spends $25.

Write down, in terms of x, an expression for the number of small sacks which Rula buys.

Answer(b) .... [1]

(c) Rula buys 4 more sacks than Paul.
Write down an equation in x and show that it simplifies to $2x^2 - 3x - 27 = 0$.

Answer(c)

[4]

(d) Solve $2x^2 - 3x - 27 = 0$.

Answer(d) $x =$ .... or $x =$ .... [3]

(e) Calculate the number of sacks which Paul buys.

Answer(e) .... [1]

10 (a) Write as a single fraction

(i) $\dfrac{5}{4} - \dfrac{2x}{5}$,

Answer(a)(i) .... [2]

(ii) $\dfrac{4}{x+3} + \dfrac{2x-1}{3}$.

Answer(a)(ii) .... [3]

(b) Solve the simultaneous equations.

$$9x - 2y = 12$$
$$3x + 4y = -10$$

Answer(b) $x =$ ....

$y =$ .... [3]

(c) Simplify $\dfrac{7x + 21}{2x^2 + 9x + 9}$.

Answer(c) .... [4]

May/June 2013 (43)

3   (a)   Luk wants to buy x goats and y sheep.

     (i)   He wants to buy at least 5 goats.

       Write down an inequality in x to represent this condition.

           Answer(a)(i) .... [1]

     (ii)   He wants to buy at least 11 sheep.

       Write down an inequality in y to represent this condition.

           Answer(a)(ii) .... [1]

     (iii)   He wants to buy at least 20 animals.

       Write down an inequality in x and y to represent this condition.

           Answer(a)(iii) .... [1]

   (b)   Goats cost \$4 and sheep cost \$8.
       The maximum Luk can spend is \$160.

       Write down an inequality in x and y and show that it simplifies to $x + 2y \leqslant 40$.

       Answer(b)

[1]

(c) (i) On the grid below, draw four lines to show the four inequalities and shade the **unwanted** regions.
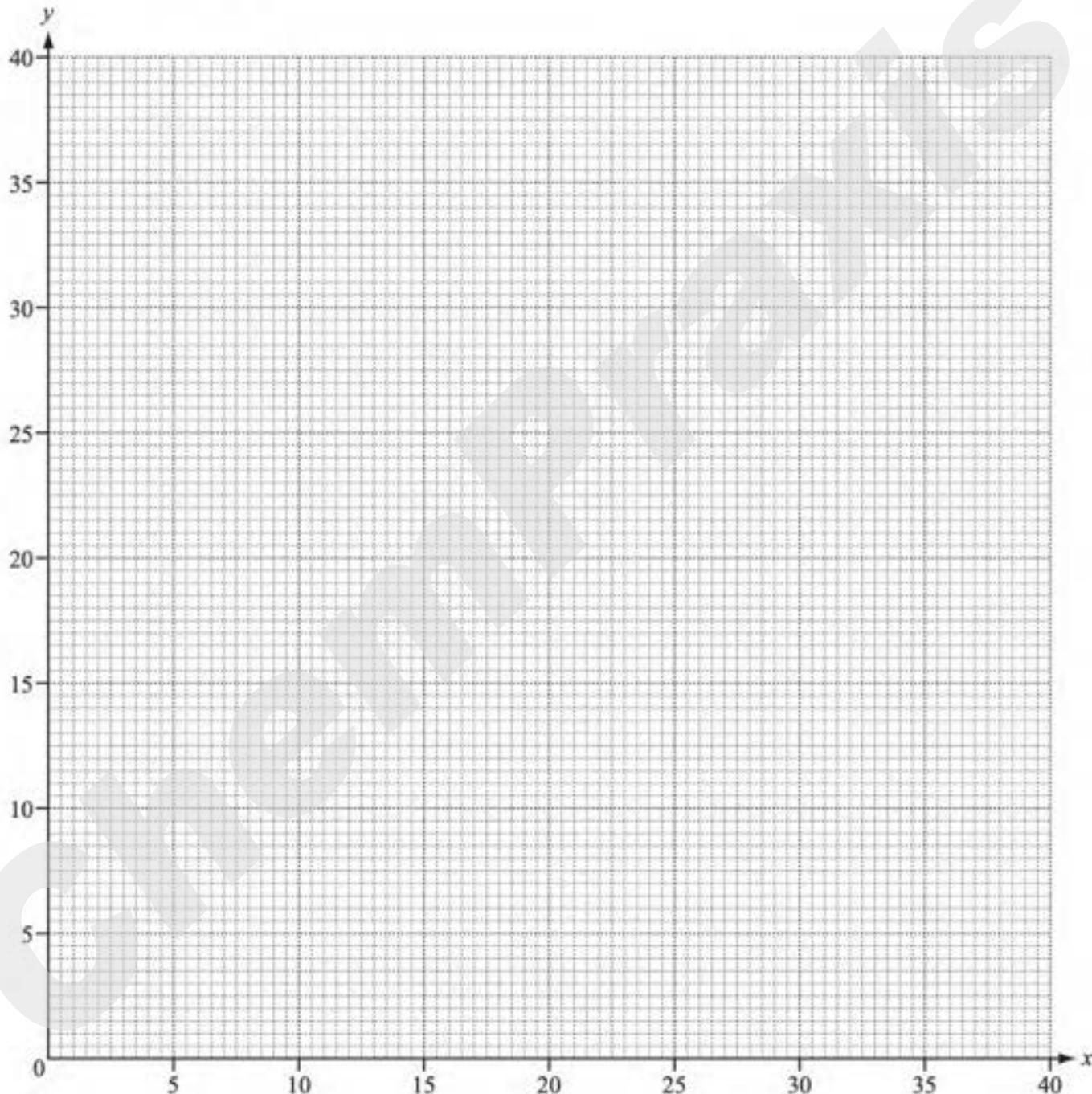
[7]

(ii) Work out the maximum number of animals that Luk can buy.

Answer(c)(ii) .... [2]

5 (a) Complete this table of values for the function $f(x) = \frac{1}{x} - x^2$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.2		0.2	0.5	1	2	3
$f(x)$	-9.33	-4.5	-2	-2.25			4.96			-3.5	-8.67

[3]

(b) Draw the graph of $f(x) = \dfrac{1}{x} - x^2$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.

[5]
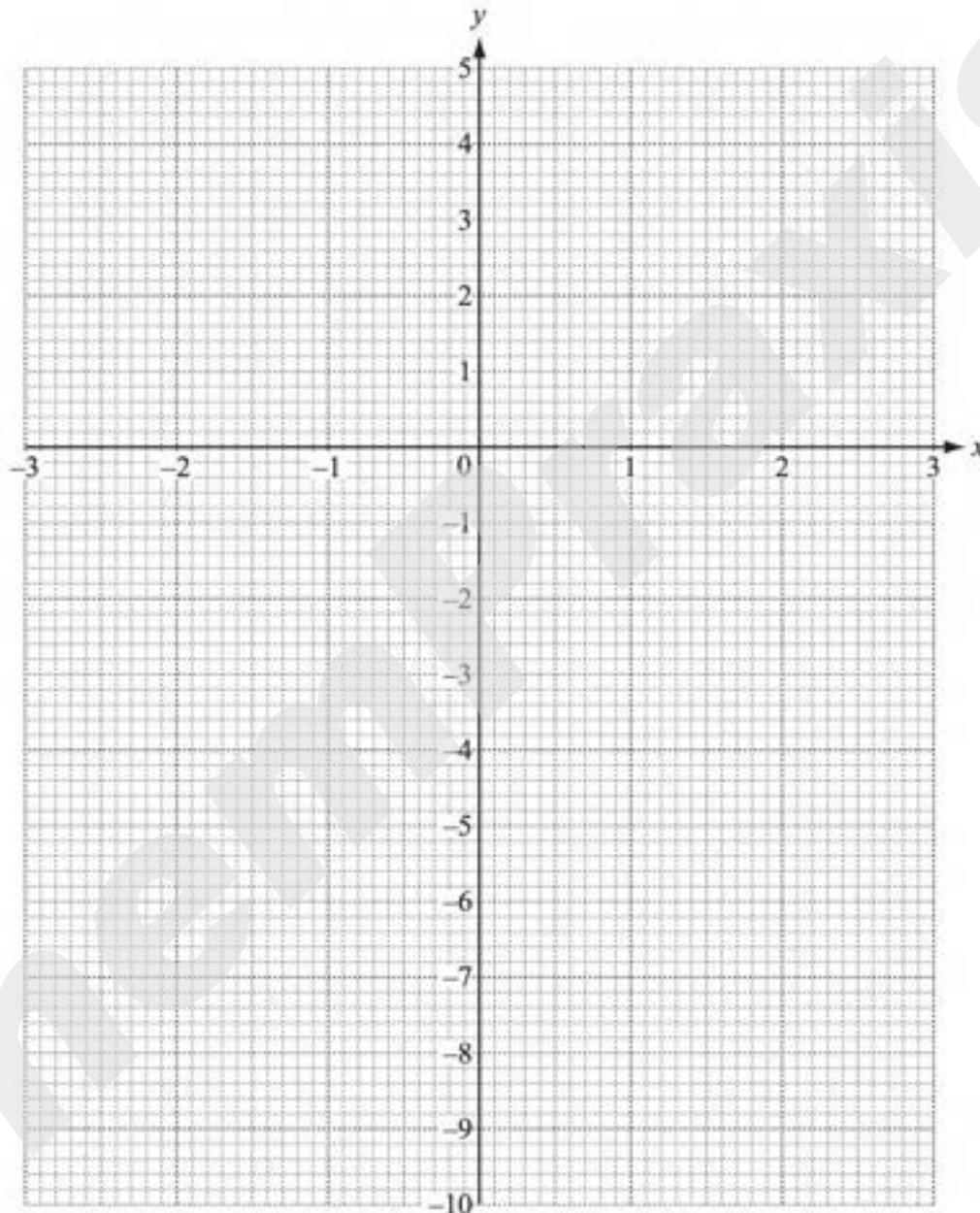
(c) Use your graph to solve $f(x) = -3$.

Answer(c) $x =$ .... or $x =$ .... or $x =$ .... [3]

(d) By drawing a suitable line on your graph, solve the equation $f(x) = 2x - 2$.

Answer(d) $x =$ .... or $x =$ .... or $x =$ .... [3]

(e) By drawing a suitable tangent, work out an estimate of the gradient of the curve at the point where $x = -2$.

You must show your working.

Answer(e) .... [3]

10 (a) (i) Solve $2(3x - 7) = 13$.

Answer(a)(i) $x =$ .... [3]

(ii) Solve by factorising $x^2 - 7x + 6 = 0$.

Answer(a)(ii) $x =$ .... or $x =$ .... [3]

(iii) Solve $\dfrac{3x-2}{5} + \dfrac{x+2}{10} = 4$.

Answer(a)(iii) $x =$ .... [4]

(b) 1^2 $= 1$

$1^2 + 2^2$ $= 5$

$1^2 + 2^2 + 3^2$ $= 14$

$1^2 + 2^2 + 3^2 + 4^2$ $= 30$

$1^2 + 2^2 + 3^2 + 4^2 + \text{............} + n^2 = an^3 + bn^2 + \dfrac{n}{6}$

Work out the values of a and b.

Answer(b) a = ...

b = ... [6]

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2   Emily cycles along a path for 2 minutes.
    She starts from rest and accelerates at a constant rate until she reaches a speed of 5 m/s after 40 seconds.
    She continues cycling at 5 m/s for 60 seconds.
    She then decelerates at a constant rate until she stops after a further 20 seconds.

    (a)   On the grid, draw a speed-time graph to show Emily's journey.
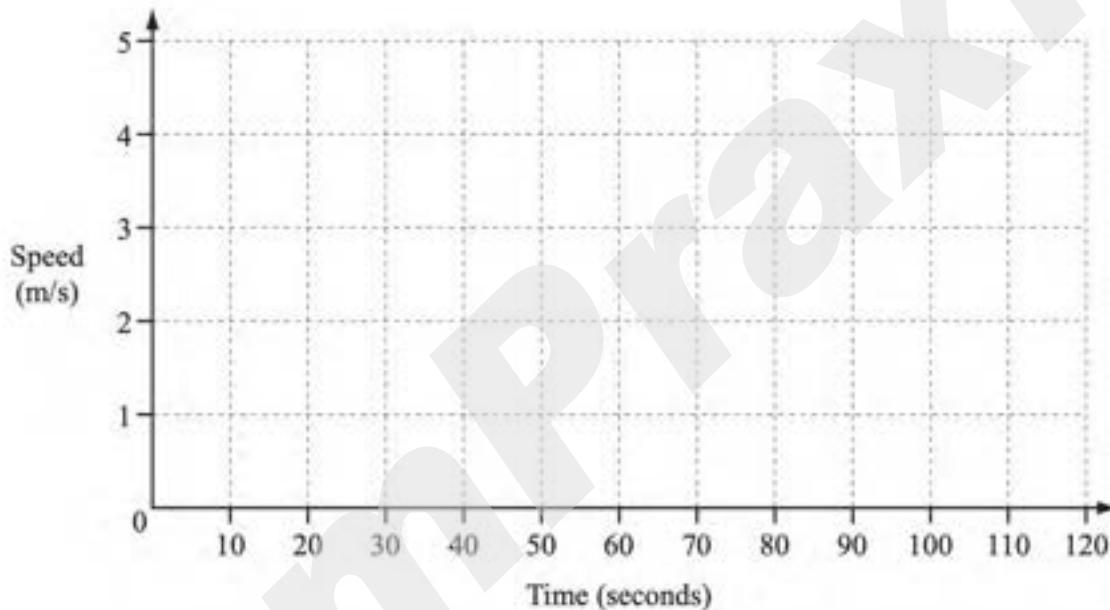

[2]

    (b)   Find Emily's acceleration.

    Answer(b) .... m/s^2 [1]

(c) Calculate Emily's average speed for the journey.

Answer(c) .... m/s [4]

6 (a)
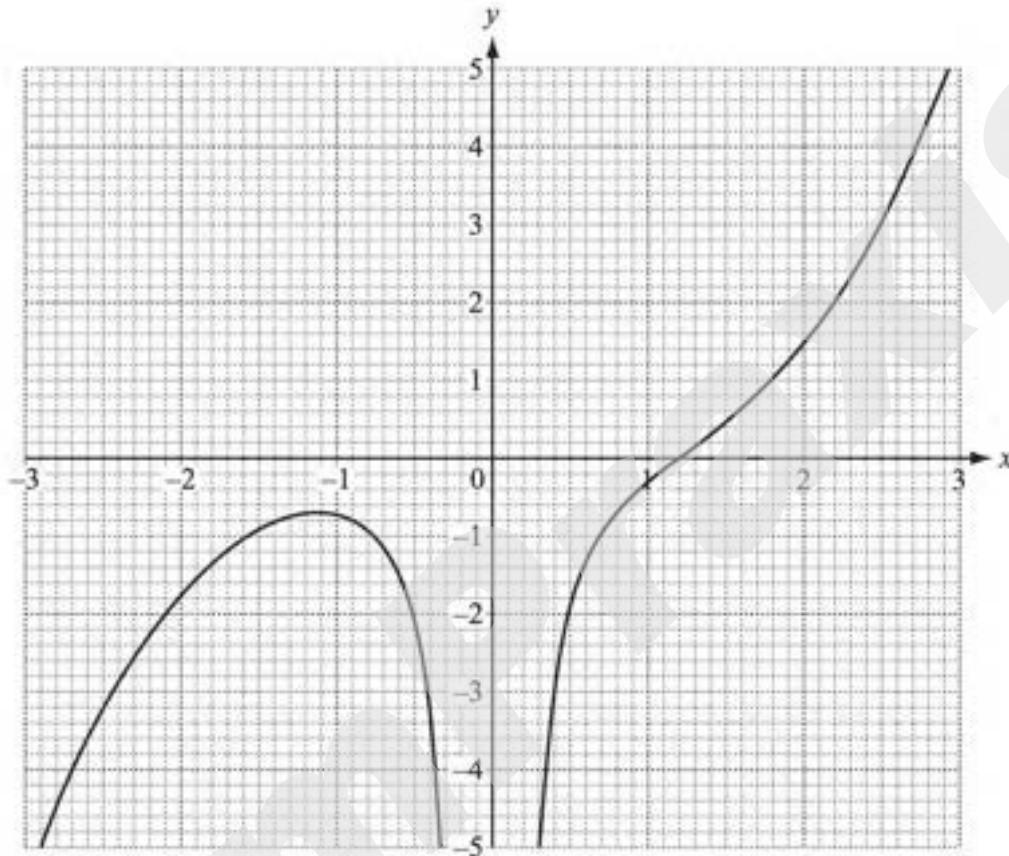
The diagram shows the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$.

(i) Find f(2).

Answer(a)(i) .... [1]

(ii) Solve the equation $f(x) = 0$.

Answer(a)(ii) $x =$ .... [1]

(iii) Write down the value of the largest **integer**, k, for which the equation $f(x) = k$ has 3 solutions.

Answer(a)(iii) $k =$ .... [1]

(iv) By drawing a suitable straight line, solve the equation $f(x) = x$.

Answer(a)(iv) $x =$ .... or $x =$ .... or $x =$ .... [3]

(b) $\quad\quad g(x) = 1 - 2x \quad\quad h(x) = x^2 - 1$

(i) Find $gh(3)$.

Answer(b)(i) .... [2]

(ii) Find $g^{-1}(x)$.

Answer(b)(ii) $g^{-1}(x) =$ .... [2]

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(iii) Solve the equation $h(x) = 3$.

Answer(b)(iii) $x =$ .... or $x =$ .... [3]

(iv) Solve the equation $g(3x) = 2x$.

Answer(b)(iv) $x =$ .... [3]

8   (a)   Solve the equation $8x^2 - 11x - 11 = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer(a) $x =$ .... or $x =$ .... [4]

(b) y varies directly as the square root of x.
$y = 18$ when $x = 9$.

Find y when $x = 484$.

Answer(b) $y =$ .... [3]

(c) Sara spends $x on pens which cost $2.50 each.
She also spends $(x − 14.50) on pencils which cost $0.50 each.
The **total** of the number of pens and the number of pencils is 19.

Write down and solve an equation in x.

Answer(c) x = ... [6]

10 (a)

$$1 = 1$$

$$1 + 2 = 3$$

$$1 + 2 + 3 = 6$$

$$1 + 2 + 3 + 4 = 10$$

(i) Write down the next line of this pattern.

Answer(a)(i) .... [1]

(ii) The sum of the first n integers is $\frac{n}{k}(n+1)$.

Show that $k = 2$.

Answer(a)(ii)

[2]

(iii) Find the sum of the first 60 integers.

Answer(a)(iii) .... [1]

(iv) Find n when the sum of the first n integers is 465.

Answer(a)(iv) n = ... [2]

(v) $1 + 2 + 3 + 4 + \ldots + x = \dfrac{(n-8)(n-7)}{2}$

Write x in terms of n.

Answer(a)(v) $x = $ .... [1]

(b)

1^3	$= 1$
$1^3 + 2^3$	$= 9$
$1^3 + 2^3 + 3^3$	$= 36$
$1^3 + 2^3 + 3^3 + 4^3$	$= 100$

(i) Complete the statement.

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = $ .... $= ($.....................$)^2$ [2]

(ii) The sum of the first n integers is $\dfrac{n}{2}(n+1)$.

Find an expression, in terms of n, for the sum of the first n cubes.

Answer(b)(ii) .... [1]

(iii) Find the sum of the first 19 cubes.

Answer(b)(iii) .... [2]

October/November 2013 (42)

3   (a) Write as a single fraction in its simplest form.

$$\frac{2x-1}{2} - \frac{3x+1}{5}$$

Answer(a) .... [3]

(b) Expand and simplify.

$$(2x-3)^2 - 3x(x-4)$$

Answer(b) .... [4]

(c) (i) Factorise.

$$2x^2 + 5x - 3$$

Answer(c)(i) .... [2]

(ii) Simplify.

$$\frac{2x^2 + 5x - 3}{2x^2 - 18}$$

Answer(c)(ii) .... [3]

5 **(a)** Complete the table of values for $y = \dfrac{2}{x^2} - \dfrac{1}{x} - 3x$.

x	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
y	9.6		6		26.5		18.0		-2	-6	-9.1

[3]

(b) Draw the graph of $y = \dfrac{2}{x^2} - \dfrac{1}{x} - 3x$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.

[5]
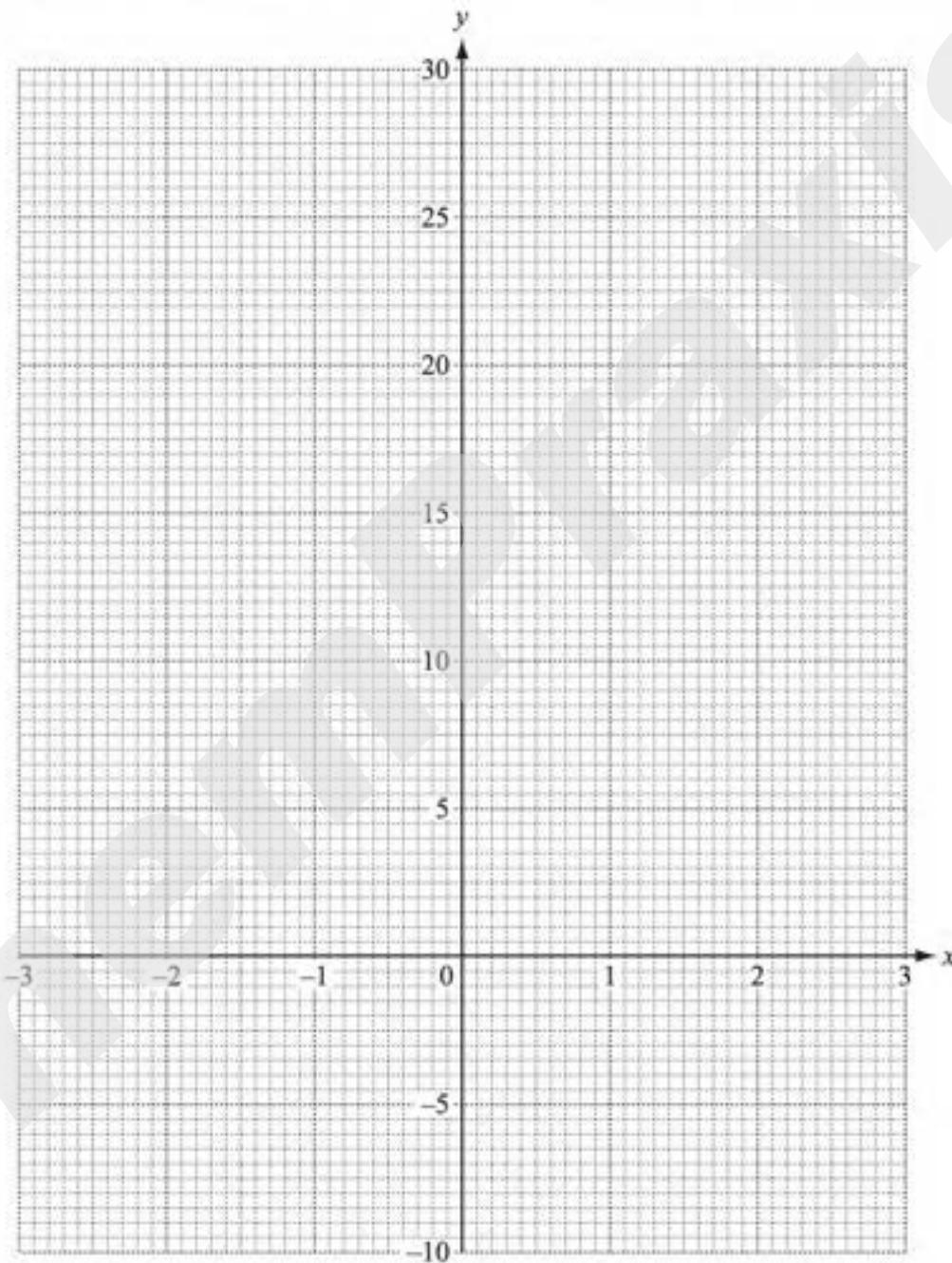
(c) Use your graph to solve these equations.

(i) $\dfrac{2}{x^2} - \dfrac{1}{x} - 3x = 0$

Answer(c)(i) $x =$ .... [1]

(ii) $\dfrac{2}{x^2} - \dfrac{1}{x} - 3x - 7.5 = 0$

Answer(c)(ii) $x =$ .... or $x =$ .... or $x =$ .... [3]

(d) (i) By drawing a suitable straight line on the graph, solve the equation $\dfrac{2}{x^2} - \dfrac{1}{x} - 3x = 10 - 3x$.

Answer(d)(i) $x =$ .... or $x =$ .... [4]

(ii) The equation $\dfrac{2}{x^2} - \dfrac{1}{x} - 3x = 10 - 3x$ can be written in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

Find the values of a, b and c.

Answer(d)(ii) $a =$ .... , $b =$ .... , $c =$ .... [3]

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8 $f(x) = 4x + 3$ $g(x) = \dfrac{7}{x+1}$ $(x \neq -1)$ $h(x) = x^2 + 5x$

(a) Work out

(i) $h(-3)$,

Answer(a)(i) .... [1]

(ii) $hg(13)$.

Answer(a)(ii) .... [2]

(b) Find $f^{-1}(x)$.

Answer(b) $f^{-1}(x) =$ .... [2]

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(c)   **(i)**   Solve the equation $f(x) = 23$.

Answer(c)(i) $x =$ .... [2]

   (ii)   Solve the equation $h(x) = 7$.

       Show all your working and give your answers correct to 2 decimal places.

Answer(c)(ii) $x =$ .... or $x =$ .... [5]

10 Complete the table for the following sequences.
The first row has been completed for you.

	Sequence				Next two terms		nth term	
	1	5	9	13	17	21	$4n - 3$	
(a)	12	21	30	39				[3]
(b)	80	74	68	62				[3]
(c)	1	8	27	64				[2]
(d)	2	10	30	68				[2]

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(c) Complete this table of values for $V = 4x^3 - 32x^2 + 63x$.

x	0	0.5	1	1.5	2	2.5	3	3.5
V	0		35	36	30		9	0

[2]

(d) On the grid opposite, draw the graph of $V = 4x^3 - 32x^2 + 63x$ for $0 \leqslant x \leqslant 3.5$.
Three of the points have been plotted for you.
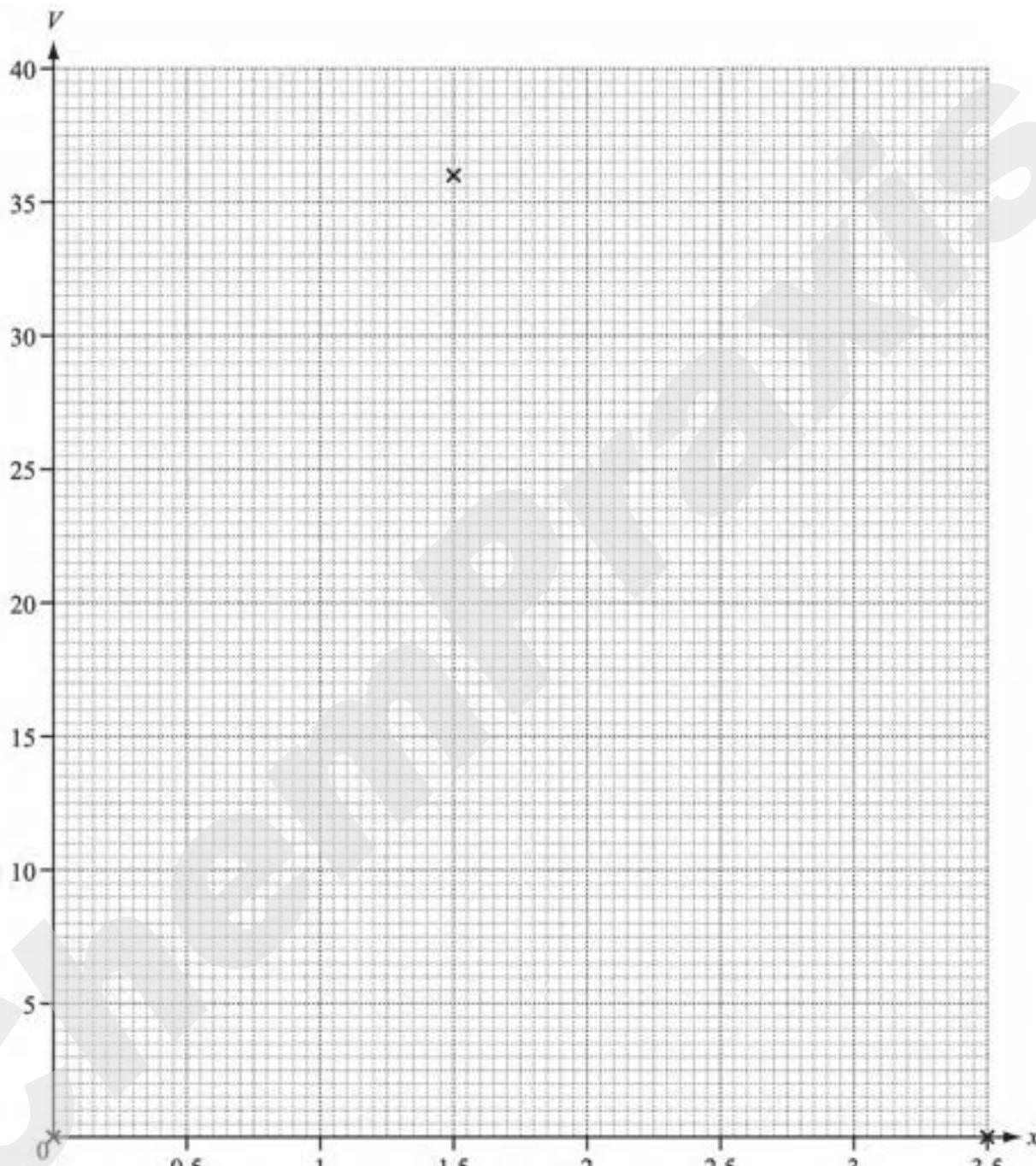
[3]

(e) The volume of the box is at least $30 \, cm^3$.
Write down, as an inequality, the possible values of x.

Answer(e) .... [2]

(f) (i) Write down the maximum volume of the box.

Answer(f)(i) .... cm^3 [1]

(ii) Write down the value of x which gives the maximum volume.

Answer(f)(ii) .... [1]

2 Ali leaves home at 10 00 to cycle to his grandmother's house. He arrives at 13 00.
The distance-time graph represents his journey.
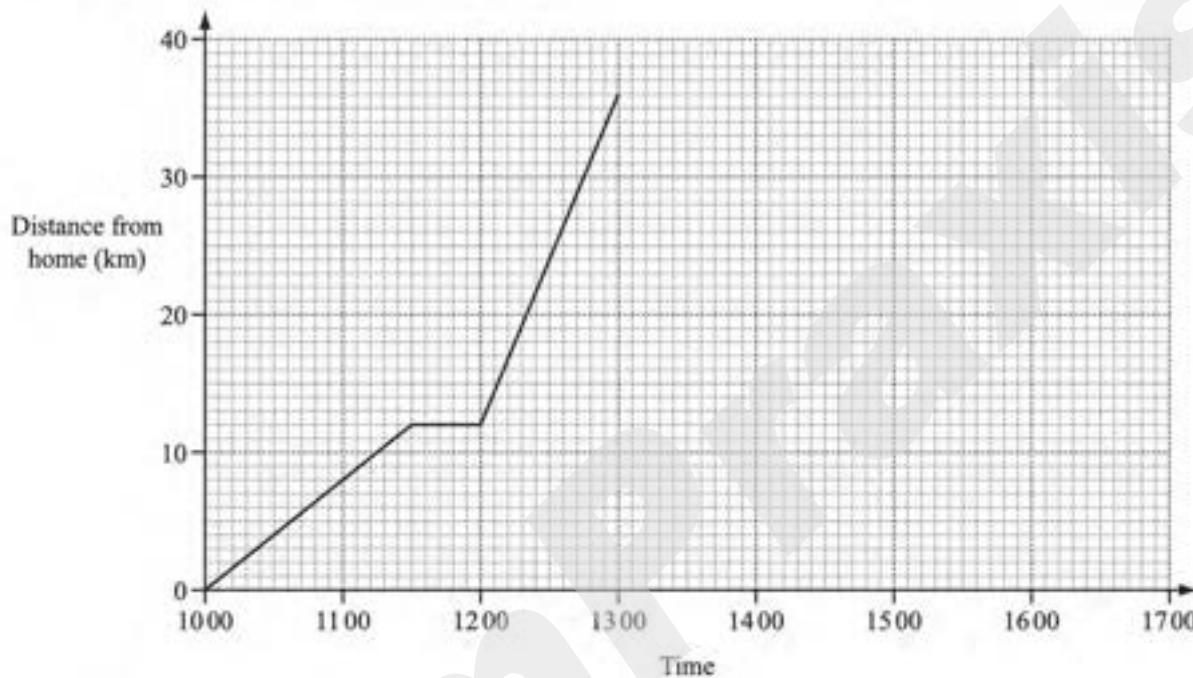
(a) Calculate Ali's speed between 10 00 and 11 30.
Give your answer in kilometres per hour.

Answer(a) .... km/h [2]

(b) Show that Ali's average speed for the whole journey to his grandmother's house is 12 km/h.

Answer(b)

[2]

(c) Change 12 kilometres per hour into metres per minute.

Answer(c) .... m/min [2]

(d) Ali stays for 45 minutes at his grandmother's house and then returns home. He arrives home at 16 42.

Complete the distance-time graph. [2]

8 (a) Complete the table of values for $y = x^3 - 3x + 1$.

x	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5
y	−7.125	−1		3		1	−0.375	−1	−0.125	3	9.125

[2]

(b) Draw the graph of $y = x^3 - 3x + 1$ for $-2.5 \leqslant x \leqslant 2.5$.
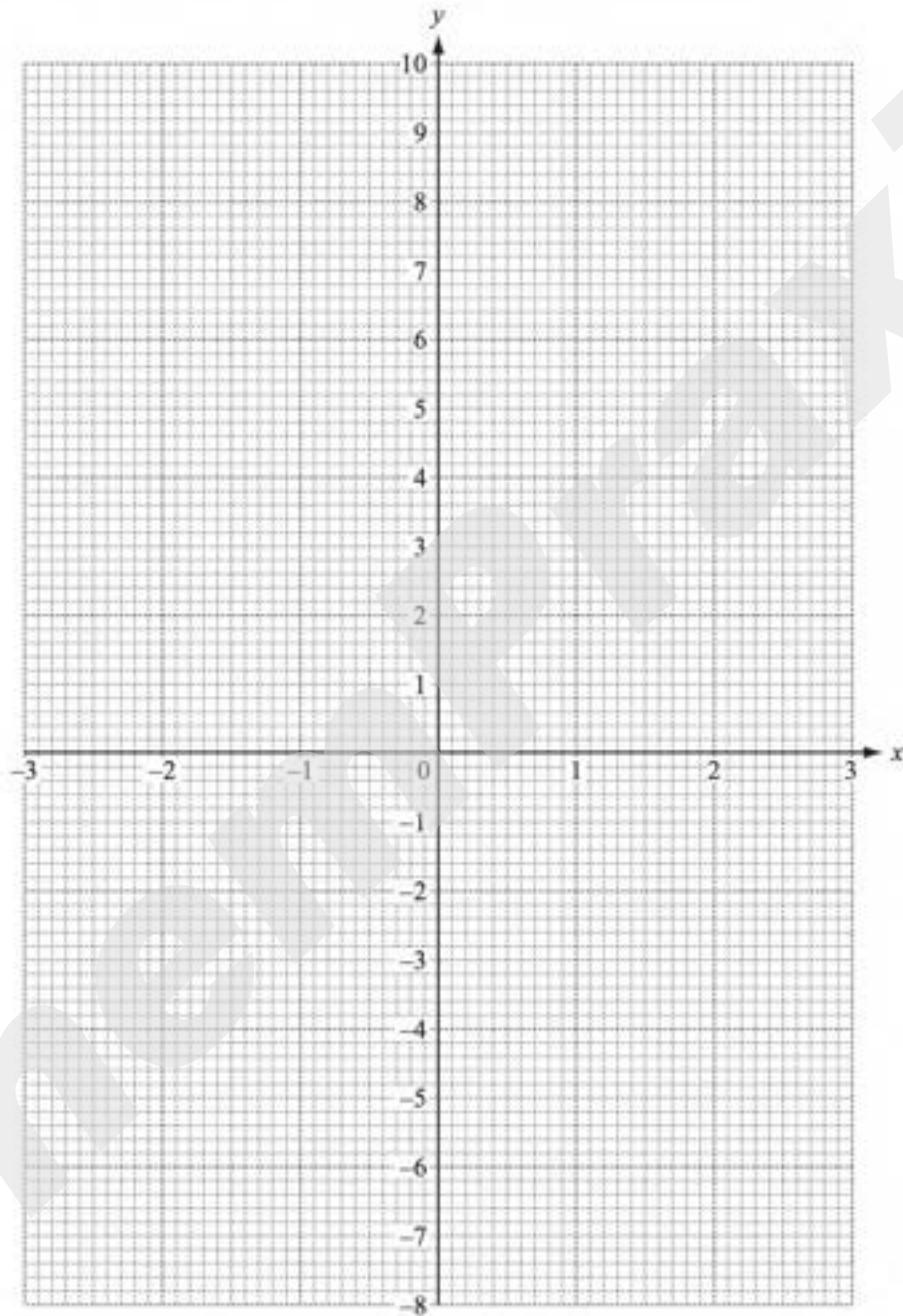
[4]

(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where $x = 2$.

Answer(c) .... [3]

(d) Use your graph to solve the equation $x^3 - 3x + 1 = 1$.

Answer(d) $x =$ .... or $x =$ .... or $x =$ .... [2]

(e) Use your graph to complete the inequality in k for which the equation

$x^3 - 3x + 1 = k$ has three different solutions.

Answer(e) .... $< k <$ .... [2]

10 (a) $f(x) = 2x - 3$ $g(x) = \dfrac{1}{x+1} + 2$ $h(x) = 3^x$

(i) Work out f(4).

Answer(a)(i) .... [1]

(ii) Work out fh(−1).

Answer(a)(ii) .... [2]

(iii)   Find $f^{-1}(x)$, the inverse of $f(x)$.

Answer(a)(iii) $f^{-1}(x) = $ .... [2]

(iv)   Find $ff(x)$ in its simplest form.

Answer(a)(iv) $ff(x) = $ .... [2]

(v)   Show that the equation $f(x) = g(x)$ simplifies to $2x^2 - 3x - 6 = 0$.

Answer(a)(v)

[3]

(vi) Solve the equation $2x^2 - 3x - 6 = 0$.

Give your answers correct to 2 decimal places.
Show all your working.

Answer(a)(vi) $x =$ _____ or $x =$ _____ [4]

(b) Simplify $\dfrac{x^2 - 3x + 2}{x^2 + 3x - 10}$.

Answer(b) _____ [4]

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2    $f(x) = \dfrac{1}{x^2} - 2x, \quad x \neq 0$

(a) Complete the table of values for f(x).

x	−3	−2.5	−2	−1.5	−1	−0.5	0.4	0.5	1	1.5	2
f(x)	6.1	5.2	4.3	3.4		5	5.5			−2.6	−3.8

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 2$.
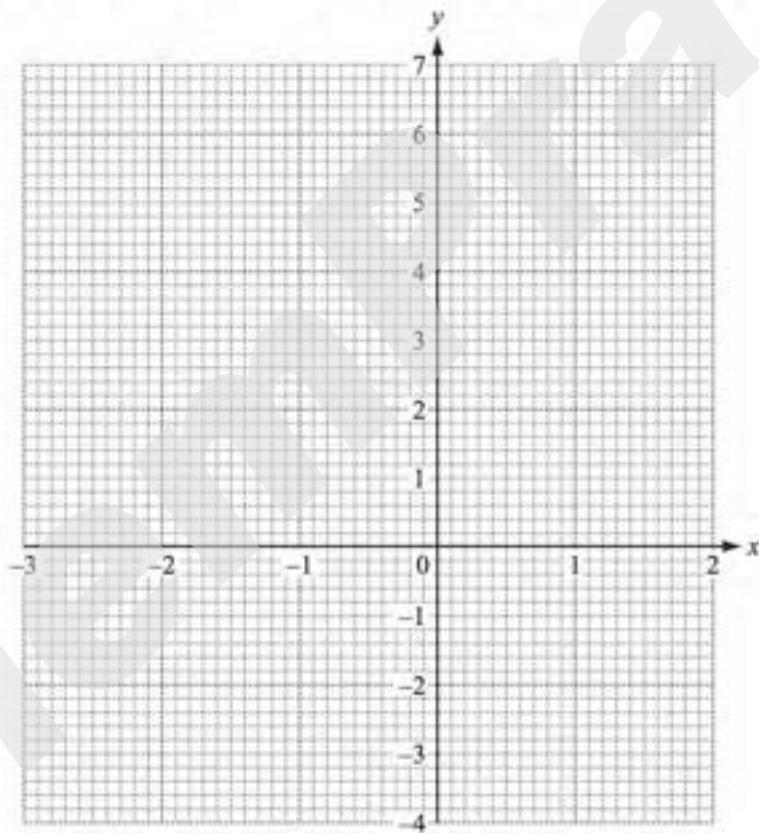
[5]

(c) Solve the equation $f(x) = 2$.

Answer(c) x = ... [1]

(d) Solve the equation $f(x) = 2x + 3$.

Answer(d) x = ... [3]

(e) (i) Draw the tangent to the graph of $y = f(x)$ at the point where $x = -1.5$. [1]

    (ii) Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

Answer(e)(ii) .... [2]

8   The distance a train travels on a journey is 600 km.

   (a) Write down an expression, in terms of x, for the average speed of the train when

      (i) the journey takes x hours,

Answer(a)(i) .... km/h [1]

      (ii) the journey takes $(x + 1)$ hours.

Answer(a)(ii) .... km/h [1]

   (b) The difference between the average speeds in **part(a)(i)** and **part(a)(ii)** is 20 km/h.

      (i) Show that $x^2 + x - 30 = 0$.

Answer(b)(i)

[3]

(ii) Find the average speed of the train for the journey in **part(a)(ii)**.
Show all your working.

Answer(b)(ii) .... km/h [4]

10 $f(x) = \dfrac{1}{x}$, $x \neq 0$ $g(x) = 1 - x$ $h(x) = x^2 + 1$

(a) Find $fg\left(\dfrac{1}{2}\right)$.

Answer(a) .... [2]

(b) Find $g^{-1}(x)$, the inverse of $g(x)$.

Answer(b) $g^{-1}(x) =$ .... [1]

(c) Find $hg(x)$, giving your answer in its simplest form.

Answer(c) $hg(x) =$ .... [3]

(d) Find the value of x when $g(x) = 7$.

Answer(d) $x =$ .... [1]

(e) Solve the equation $h(x) = 3x$.
Show your working and give your answers correct to 2 decimal places.

Answer(e) $x =$ .... or $x =$ .... [4]

(f) A function $k(x)$ is its own inverse when $k^{-1}(x) = k(x)$.

For which of the functions $f(x)$, $g(x)$ and $h(x)$ is this true?

Answer(f) .... [1]

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4 The table shows some values for the function $y = \dfrac{1}{x^2} + x$, $x \neq 0$.

x	-3	-2	-1	-0.5	0.5	1	2	3	4
y	-2.89	-1.75		3.5		2	2.25		4.06

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = \dfrac{1}{x^2} + x$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

[5]
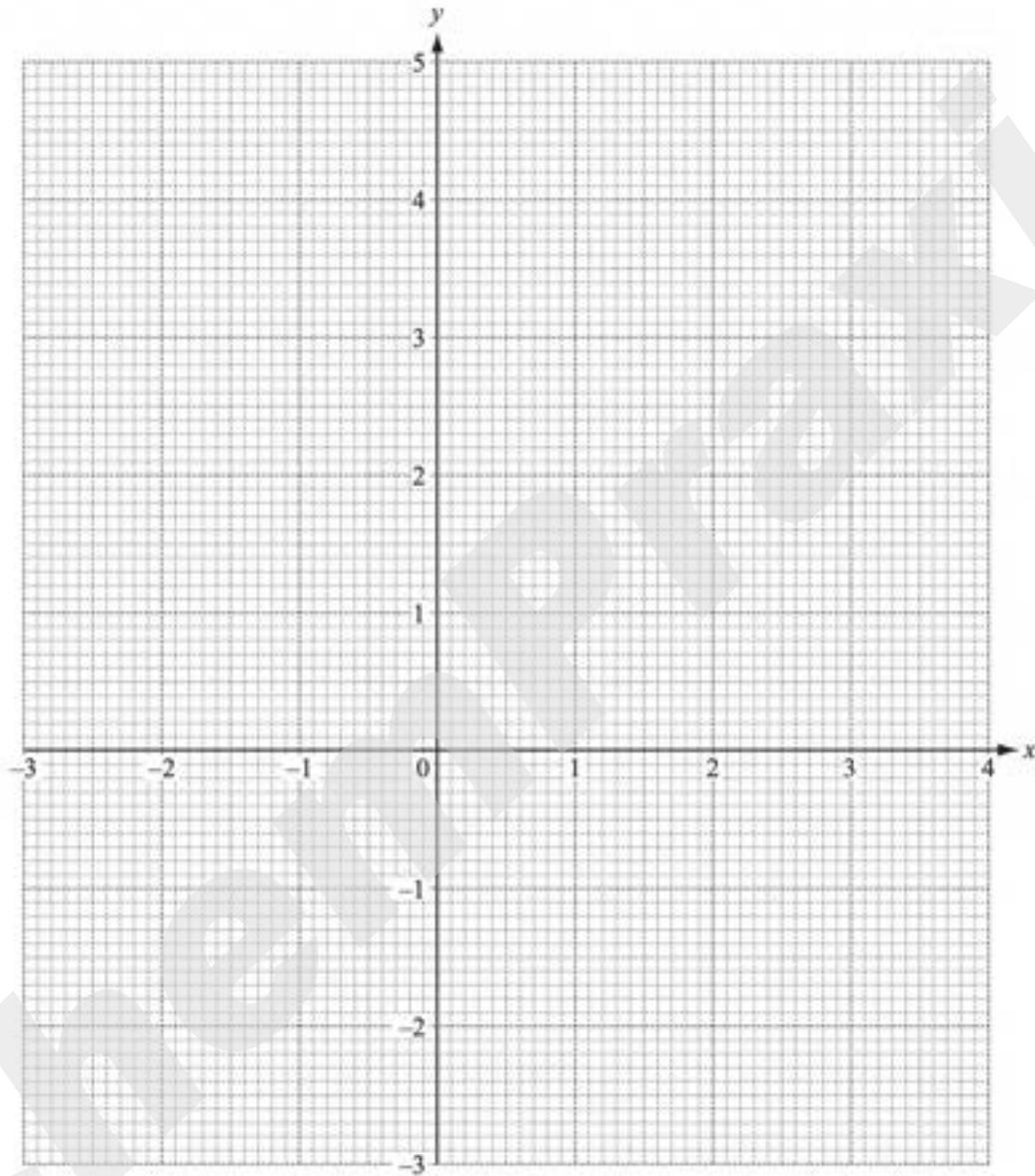
(c) Use your graph to solve the equation $\frac{1}{x^2} + x - 3 = 0$.

Answer(c) x = ... or *x =* .... or *x =* .... [3]

(d) Use your graph to solve the equation $\frac{1}{x^2} + x = 1 - x$.

Answer(d) x = ... [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

Answer(e) .... [3]

(f) Using algebra, show that you can use the graph at $y = 0$ to find $\sqrt[3]{-1}$.

Answer(f)

[3]

8 (a) (i) Show that the equation $\dfrac{7}{x+4} + \dfrac{2x-3}{2} = 1$ can be simplified to $2x^2 + 3x - 6 = 0$.

Answer(a)(i)

[3]

(ii) Solve the equation $2x^2 + 3x - 6 = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(a)(ii) x = ... *or x =* .... [4]

9      $f(x) = 4 - 3x$ $g(x) = 3^{-x}$

(a) Find $f(2x)$ in terms of x.

Answer(a) f(2x) = ... [1]

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(b) Find ff(x) in its simplest form.

Answer(b) ff(x) = ... [2]

(c) Work out gg(-1).
Give your answer as a fraction.

Answer(c) .... [3]

(d) Find f^{-1}(x), the inverse of f(x).

Answer(d) f^{-1}(x) = ... [2]

(e) Solve the equation gf(x) = 1.

Answer(e) x = ... [3]

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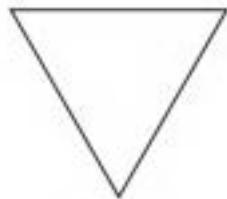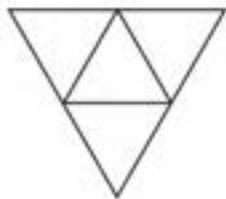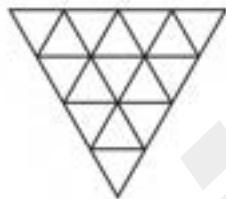
| Diagram 1	Diagram 2	Diagram 3

The first three diagrams in a sequence are shown above.
Diagram 1 shows an equilateral triangle with sides of length 1 unit.

In Diagram 2, there are 4 triangles with sides of length $\frac{1}{2}$ unit.

In Diagram 3, there are 16 triangles with sides of length $\frac{1}{4}$ unit.

(a) Complete this table for Diagrams 4, 5, 6 and n.

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6	Diagram n
Length of side	1	$\frac{1}{2}$	$\frac{1}{4}$				
Length of side as a power of 2	2^0	2^{-1}	2^{-2}				

[6]

(b) (i) Complete this table for the number of the smallest triangles in Diagrams 4, 5 and 6.

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5	Diagram 6
Number of smallest triangles	1	4	16			
Number of smallest triangles as a power of 2	2^0	2^2	2^4			

[2]

(ii) Find the number of the smallest triangles in Diagram n, giving your answer as a power of 2.

Answer(b)(ii) .... [1]

(c) Calculate the number of the smallest triangles in the diagram where the smallest triangles have sides of length $\frac{1}{128}$ unit.

Answer(c) .... [2]

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2 **(a)** Rearrange the formula $v^2 = u^2 - 2as$ to make u the subject.

Answer(a) $u =$ .... [2]

(b) Chuck cycles along Skyline Drive.
He cycles 60 km at an average speed of x km/h.
He then cycles a further 45 km at an average speed of $(x + 4)$ km/h.
His total journey time is 6 hours.

    (i) Write down an equation in x and show that it simplifies to $2x^2 - 27x - 80 = 0$.

      Answer(b)(i)

                                                                     [4]

    (ii) Solve $2x^2 - 27x - 80 = 0$ to find the value of x.

                                        Answer(b)(ii) $x =$ .... [3]

(c) The diagram shows the speed-time graph for a car travelling along a road for T seconds.
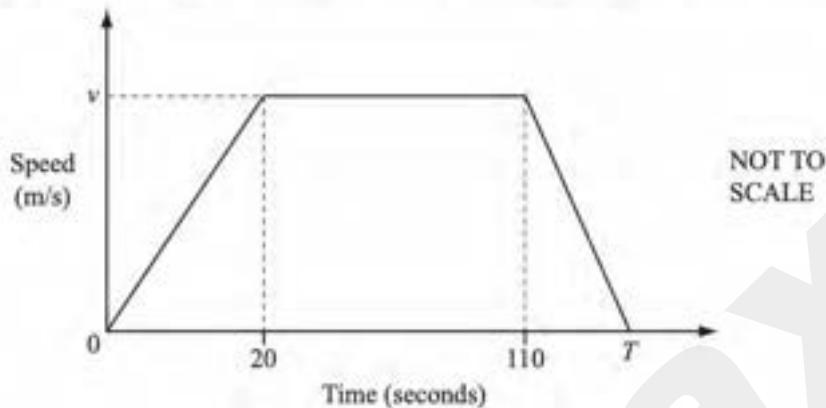
To begin with the car accelerated at $0.75 \, m/s^2$ for 20 seconds to reach a speed of $v \, m/s$.

(i) Show that the speed, v, of the car is $15 \, m/s$.

Answer(c)(i)

[1]

(ii) The total distance travelled is 1.8 **kilometres**.

Calculate the total time, T, of the journey.

Answer(c)(ii) .... seconds [4]

4 (a) Expand and simplify.

(i) $4(2x - 1) - 3(3x - 5)$

Answer(a)(i) .... [2]

(ii) $(2x - 3y)(3x + 4y)$

Answer(a)(ii) .... [3]

(b) Factorise.

$x^3 - 5x$

Answer(b) .... [1]

(c) Solve the inequality.

$$\frac{2x + 1}{3} \leq \frac{5x - 8}{4}$$

Answer(c) .... [3]

(d) (i) $x^2 - 9x + 12 = (x-p)^2 - q$

Find the value of p and the value of q.

Answer(d)(i) $p =$ ....

$q =$ .... [3]

(ii) Write down the minimum value of $x^2 - 9x + 12$.

Answer(d)(ii) .... [1]

(iii) Write down the equation of the line of symmetry of the graph of $y = x^2 - 9x + 12$.

Answer(d)(iii) .... [1]

5 (a) Complete the table of values for $y = x^2 + \dfrac{3}{x}$, $x \neq 0$.

x	-3	-2	-1	-0.5		0.4	0.6	1	1.5	2	3
y	8	2.5		-5.8		7.7	5.4	4	4.3		10

[2]

(b) Draw the graph of $y = x^2 + \frac{3}{x}$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 3$.

[5]
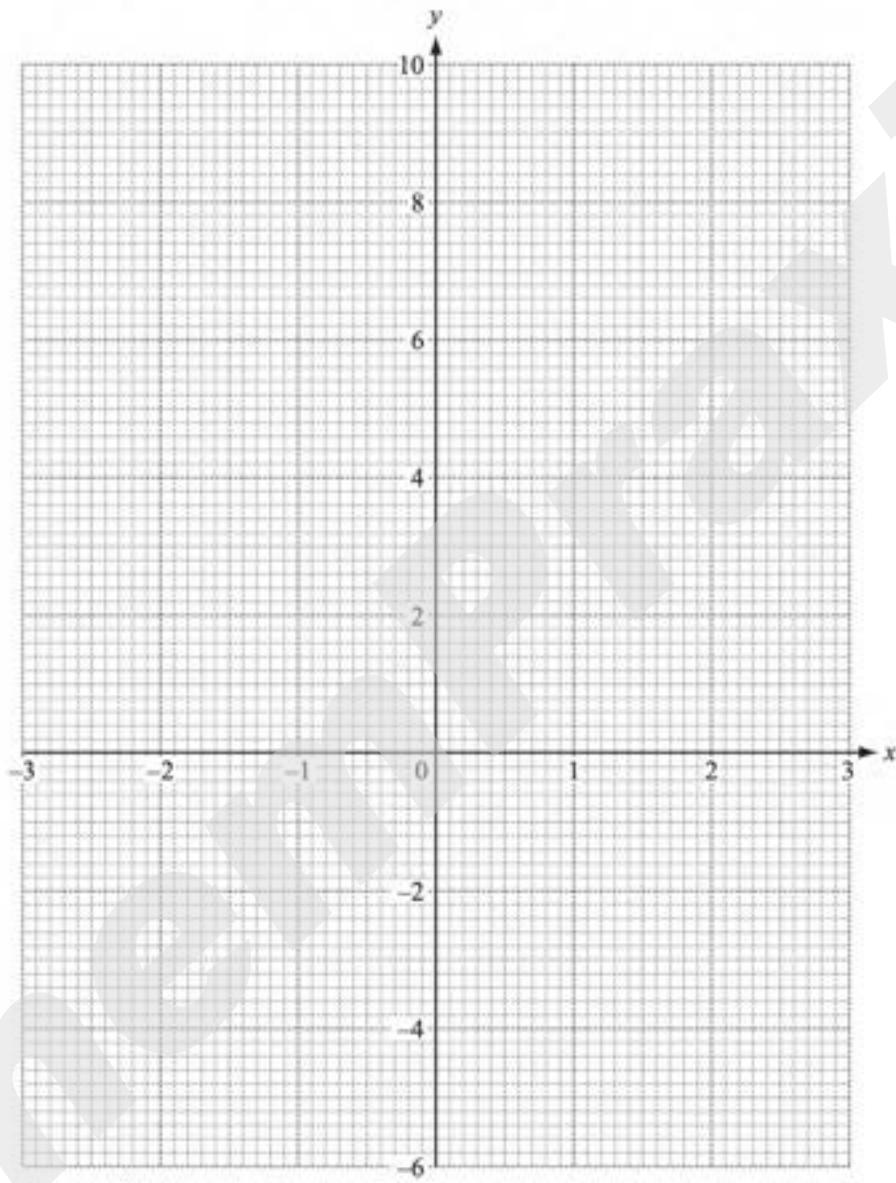
(c) Use your graph to solve the equation $x^2 + \frac{3}{x} = 5$.

Answer(c) $x =$ .... or $x =$ .... or $x =$ .... [3]

(d) By drawing a suitable straight line, solve the equation $x^2 + \frac{3}{x} = x + 5$.

Answer(d) $x =$ .... or $x =$ .... or $x =$ .... [4]

9
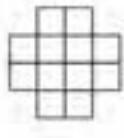
Diagram 1   Diagram 2   Diagram 3   Diagram 4
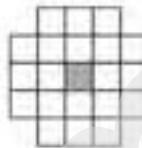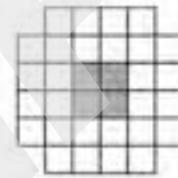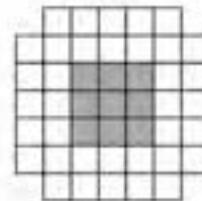
The first four diagrams in a sequence are shown above.
The diagrams are drawn using white squares ☐ and grey squares ■.

(a) Complete the columns in the table for Diagram 4 and Diagram n.

Diagram	1	2	3	4		n
Number of white squares	12	20	28			
Number of grey squares	0	1	4			
Total number of squares	12	21	32			$(n+1)(n+5)$

[6]

(b) Work out the number of the diagram which has a total of 480 squares.

Answer(b) .... [2]

(c) The **total** number of squares in the **first *n* diagrams** is

$$\frac{1}{3}n^3 + pn^2 + qn.$$

(i) Use $n = 1$ in this expression to show that $p + q = 11\frac{2}{3}$.

Answer(c)(i)

[1]

(ii) Use $n = 2$ in the expression to show that $4p + 2q = 30\frac{1}{3}$.

Answer(c)(ii)

[2]

(iii) Find the values of p and q.

Answer(c)(iii) $p =$ ....

$q =$ .... [3]

October/November 2014 (42)

2 **(a)** Solve the inequality.

$$7x - 5 > 3(2 - 5x)$$

Answer(a) .... [3]

(b) **(i)** Factorise completely.

$$pq - 2q - 8 + 4p$$

Answer(b)(i) .... [2]

(ii) Factorise.

$$9p^2 - 25$$

Answer(b)(ii) .... [1]

(c) Solve this equation by factorising.

$$5x^2 + x - 18 = 0$$

Answer(c) $x =$ .... or $x =$ .... [3]

5      $f(x) = 5x - 2$ $g(x) = \dfrac{7}{x-3}$, $x \neq 3$ $h(x) = 2x^2 + 7x$

(a) Work out

   (i) $f(2)$.

Answer(a)(i) .... [1]

   (ii) $hg(17)$.

Answer(a)(ii) .... [2]

(b) Solve $g(x) = x + 3$.

Answer(b) $x =$ .... or $x =$ .... [3]

(c) Solve $h(x) = 11$, showing all your working and giving your answers correct to 2 decimal places.

Answer(c) $x =$ .... or $x =$ .... [5]

(d) Find $f^{-1}(x)$.

Answer(d) $f^{-1}(x) =$ .... [2]

(e) Solve $g^{-1}(x) = -0.5$.

Answer(e) $x =$ .... [1]

6    $f(x) = 5x^3 - 8x^2 + 10$

(a) Complete the table of values.

x	−1.5	−1	−0.5	0	0.5	0.75	1	1.5	2
$f(x)$	−24.9			10	8.6	7.6	7		18

[3]

(b) Draw the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 2$.

[4]
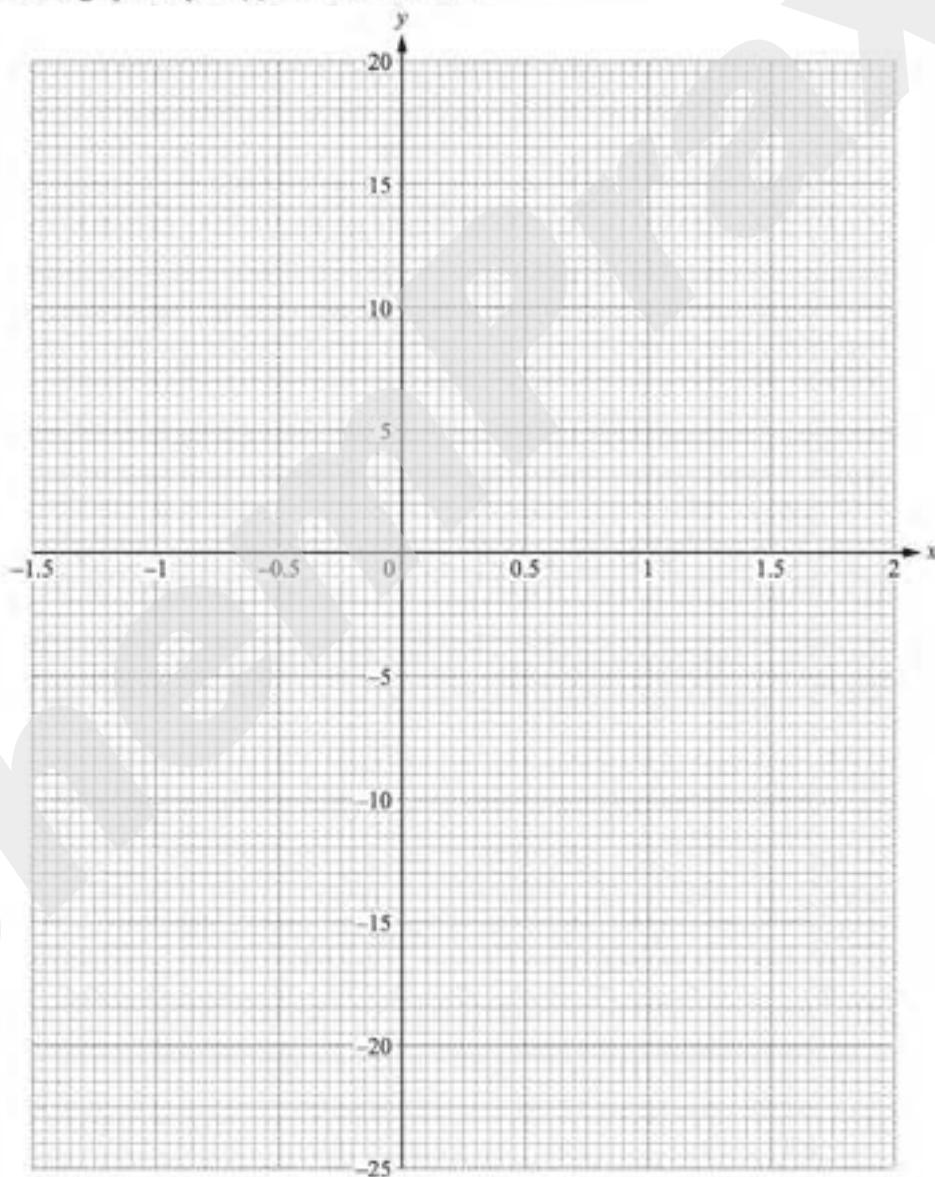
(c) Use your graph to find an **integer** value of k so that $f(x) = k$ has

    (i) exactly one solution,

Answer(c)(i) k = ... [1]

    (ii) three solutions.

Answer(c)(ii) k = ... [1]

(d) By drawing a suitable straight line on the graph, solve the equation $f(x) = 15x + 2$ for $-1.5 \leqslant x \leqslant 2$.

Answer(d) x = ... or *x =* .... [4]

(e) Draw a tangent to the graph of $y = f(x)$ at the point where $x = 1.5$.

Use your tangent to estimate the gradient of $y = f(x)$ when $x = 1.5$.

Answer(e) .... [3]

9

Layer 1

Layer 2

Layer 3
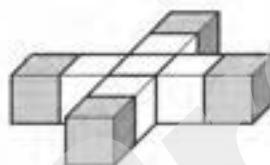
The diagrams show layers of white and grey cubes.
Khadega places these layers on top of each other to make a tower.

(a) Complete the table for towers with 5 and 6 layers.

Number of layers	1	2	3	4	5	6
Total number of **white** cubes	0	1	6	15		
Total number of grey cubes	1	5	9	13		
Total number of cubes	1	6	15	28		

[4]

(b) **(i)** Find, in terms of n, the **total** number of **grey** cubes in a tower with n layers.

Answer(b)(i) .... [2]

(ii) Find the total number of grey cubes in a tower with 60 layers.

Answer(b)(ii) .... [1]

(iii) Khadega has plenty of white cubes but only 200 grey cubes.
How many layers are there in the highest tower that she can build?

Answer(b)(iii) .... [2]

(c) The expression for the **total** number of **white** cubes in a tower with n layers is $pn^2 + qn + 3$.

Find the value of p and the value of q.
Show all your working.

Answer(c) p = ...

$q =$ .... [5]

(d) Find an expression, in terms of n, for the **total** number of cubes in a tower with n layers.
Give your answer in its simplest form.

Answer(d) .... [2]

6 (a) Simplify.

   (i) $x^3 \div \dfrac{3}{x^5}$

   Answer(a)(i) .... [1]

   (ii) $5xy^8 \times 3x^6y^{-5}$

   Answer(a)(ii) .... [2]

   (iii) $(64x^{12})^{\frac{1}{3}}$

   Answer(a)(iii) .... [2]

(b) Solve $3x^2 - 7x - 12 = 0$.
     Show your working and give your answers correct to 2 decimal places.

   Answer(b) x = ... *or x =* .... [4]

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Question 8c and 8d

(c) Sketch the graph of $y = 18 + 7x - x^2$ on the axes below.
Indicate clearly the values where the graph crosses the x and y axes.

NOT TO
SCALE
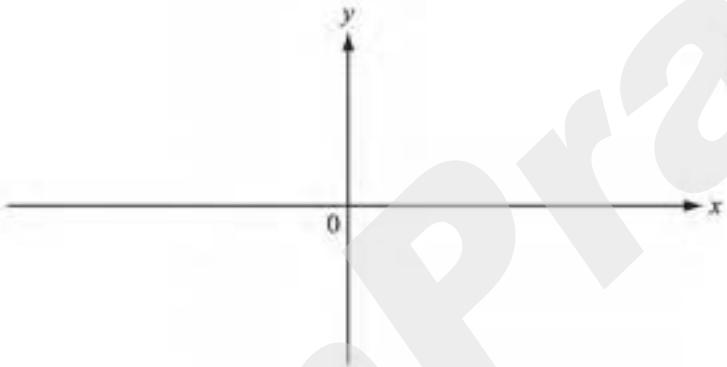
[4]

(d) (i) $x^2 + 12x - 7 = (x + p)^2 - q$

Find the value of p and the value of q.

Answer(d)(i) $p =$ ....

$q =$ .... [3]

(ii) Write down the minimum value of y for the graph of $y = x^2 + 12x - 7$.

Answer(d)(ii) .... [1]

10 (a)

$(3x - 5)$ cm

$(2x - 3)$ cm

NOT TO
SCALE

$(15 - 2x)$ cm

$(2x + 7)$ cm
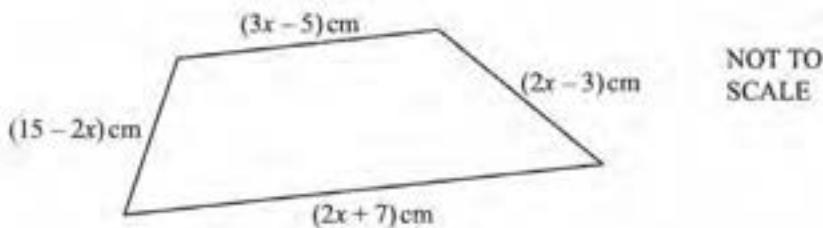
(i)   Write an expression, in terms of x, for the perimeter of the quadrilateral.
      Give your answer in its simplest form.

Answer(a)(i) .... cm [2]

(ii) The perimeter of the quadrilateral is 32 cm.

      Find the length of the longest side of the quadrilateral.

Answer(a)(ii) .... cm [3]

(b)
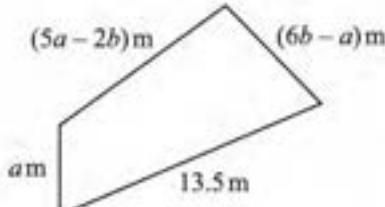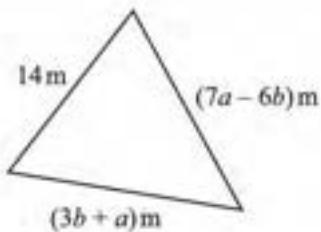
The triangle has a perimeter of 32.5 m.
The quadrilateral has a perimeter of 39.75 m.

Write two equations in terms of a and b and simplify them.
Use an algebraic method to find the values of a and b.
Show all your working.

Answer(b) a = ...

b = ... [6]

February/March 2015 (42)

3   The table shows some values of $y = x^3 + 3x^2 - 2$.

x	–3	–2.5	–2	–1.5	–1	–0.5	0	0.5	1
y	–2	1.13		1.38		–1.38		–1.13	

(a) Complete the table of values.   [3]

(b) On the grid, draw the graph of $y = x^3 + 3x^2 - 2$ for $-3 \leqslant x \leqslant 1$.

[4]
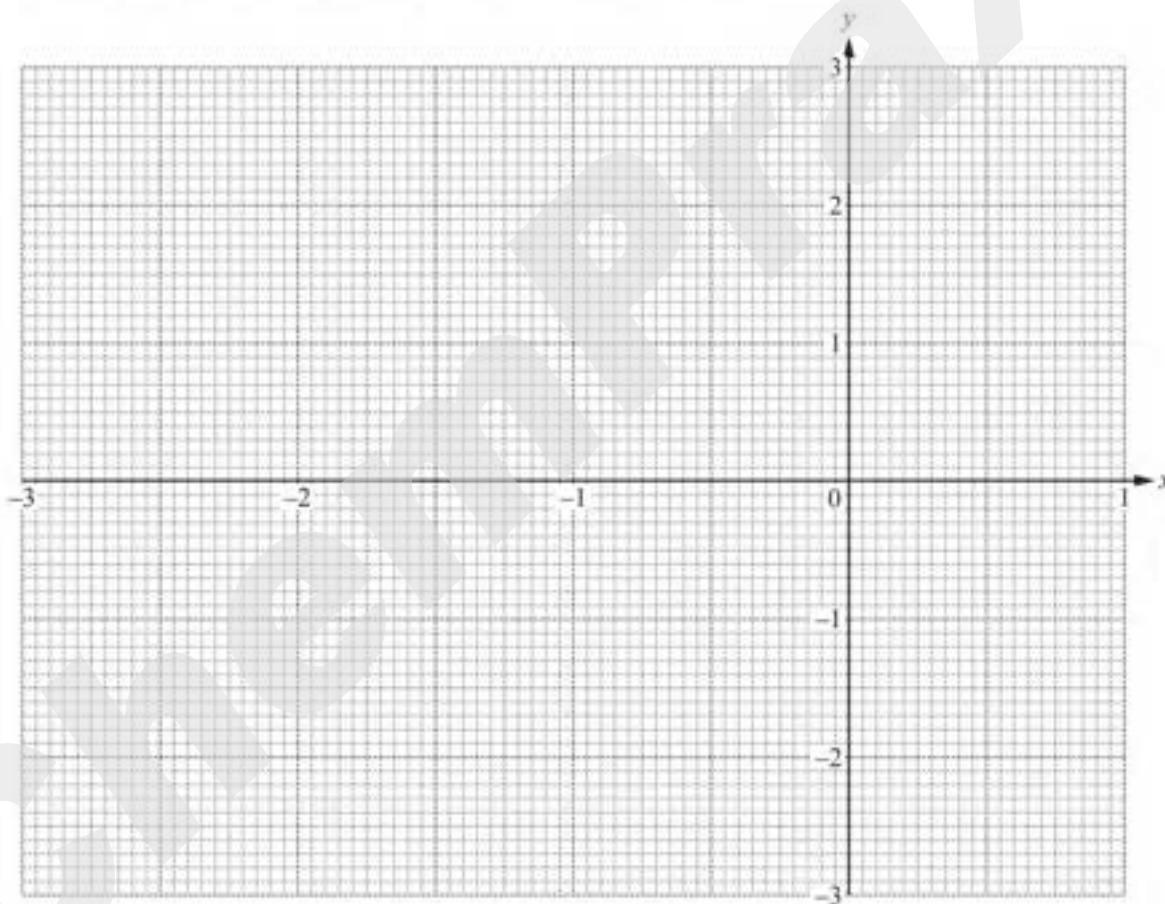
(c) By drawing a suitable line, solve the equation $x^3 + 3x^2 - 2 = \frac{1}{2}(x + 1)$.

Answer(c) x = ... or $x =$ .... or $x =$ .... [4]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.75$.

Answer(d) .... [3]

4 (a) Factorise $121y^2 - m^2$.

Answer(a) .... [2]

(b) Write as a single fraction in its simplest form.

$$\frac{4}{3x - 5} + \frac{x + 2}{x - 1}$$

Answer(b) .... [3]

(c) Solve the equation.

$$3x^2 + 2x - 7 = 0$$

Show all your working and give your answers correct to 2 decimal places.

Answer(c) x = ... *or x =* .... [4]

(d) In this part, all lengths are in centimetres.

NOT TO SCALE
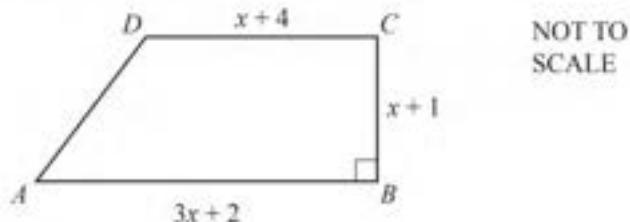
$ABCD$ is a trapezium with area $15\,\text{cm}^2$.

(ii) Solve the equation $2x^2 + 5x - 12 = 0$.

Answer(d)(ii) $x =$ .... or $x =$ .... [3]

(iii) Write down the length of AB.

Answer(d)(iii) $AB =$ .... cm [1]

10 The school cook buys potatoes in small sacks, each of mass 4 kg, and large sacks, each of mass 10 kg.
He buys x small sacks and y large sacks.
Today, he buys less than 80 kg of potatoes.

(a) Show that $2x + 5y < 40$.

Answer(a)

[1]

(b) He buys more large sacks than small sacks.
He buys no more than 6 large sacks.

Write down two inequalities to show this information.

Answer(b) ....

.... [2]

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(c) On the grid, show the information in **part (a)** and **part (b)** by drawing three straight lines and shading the unwanted regions.
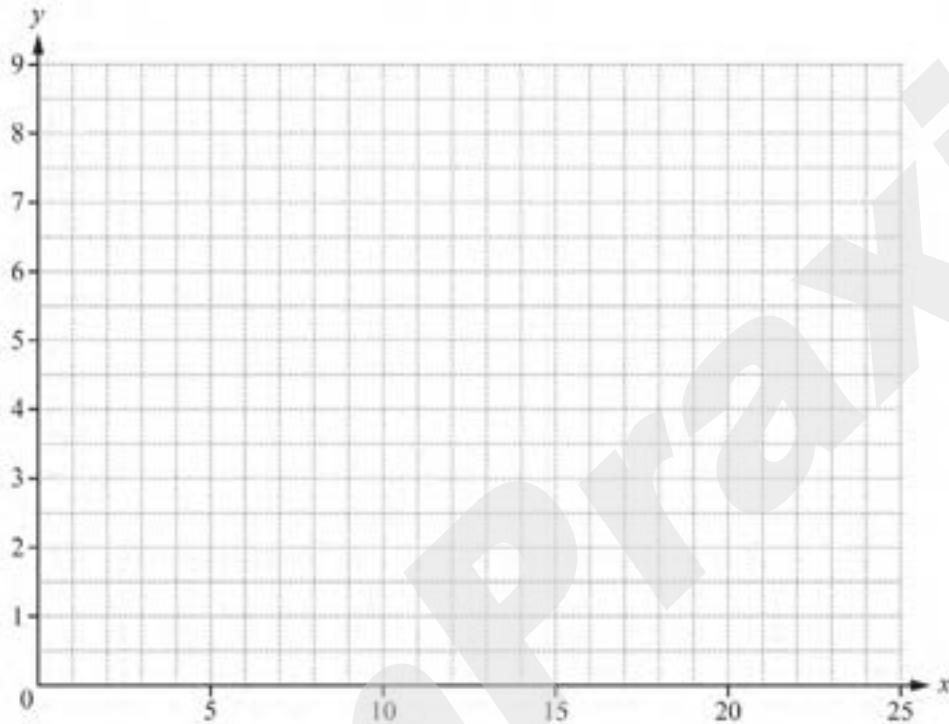
[5]

(d) Find the greatest mass of potatoes the cook can buy today.

Answer(d) .... kg [2]

11

Diagram 1   Diagram 2   Diagram 3   Diagram 4   Diagram 5
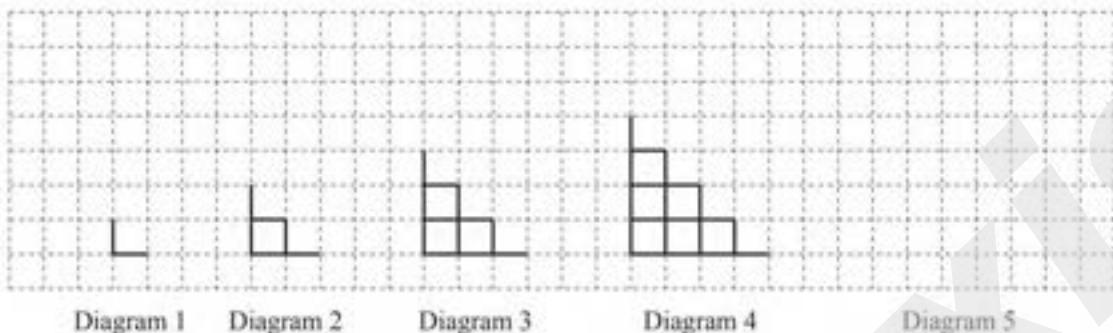
Diagram 1 shows two lines of length 1 unit at right angles forming an ⌐.

Two ⌐s are added to Diagram 1 to make Diagram 2. This forms one small square.

Three ⌐s are added to Diagram 2 to make Diagram 3. This forms three small squares.
The sequence of Diagrams continues.

(a) Draw Diagram 5. [1]

(b) Complete the table.

	Diagram 1	Diagram 2	Diagram 3	Diagram 4	Diagram 5
Number of lines of length 1 unit	2	6	12	20	
Number of small squares	0	1	3	6	

[2]

(c) Find an expression, in terms of n, for the number of lines of length 1 unit in Diagram n.

Answer(c) .... [2]

(d) Find an expression, in terms of n, for the number of small squares in Diagram n.

Answer(d) .... [2]

May/June 2015 (41)

2 The table shows some values for $y = x^2 - \frac{1}{2x}, x \neq 0$.

x	-2	-1.5	-1	-0.5	-0.25	-0.2		0.2	0.25	0.5	1	1.5	2
y	4.25	2.58			2.06	2.54		-2.46	-1.94			1.92	3.75

(a) Complete the table of values.   [4]

(b) On the grid, draw the graph of $y = x^2 - \frac{1}{2x}$ for $-2 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 2$.

[5]
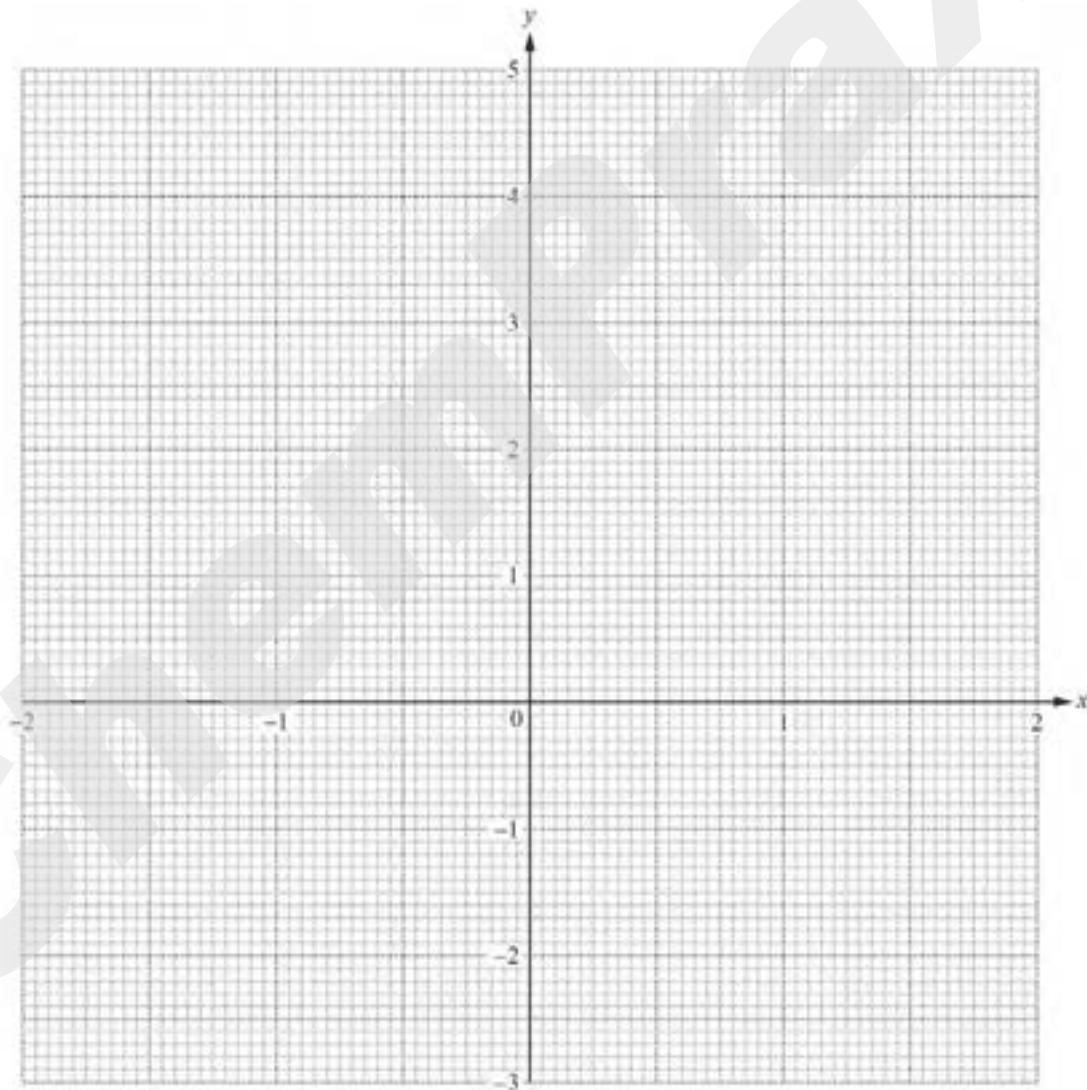
(c) By drawing a suitable line, use your graph to solve the equation $x^2 - \frac{1}{2x} = 2$.

Answer(c) $x =$ .... or $x =$ .... or $x =$ .... [3]

(d) The equation $x^2 - \frac{1}{2x} = k$ has only one solution.

Write down the range of values of k for which this is possible.

Answer(d) .... [2]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1$.

Answer(e) .... [3]

8   (a) Jamil, Kiera and Luther collect badges.
        Jamil has x badges.
        Kiera has 12 badges more than Jamil.
        Luther has 3 times as many badges as Kiera.
        Altogether they have 123 badges.

        Form an equation and solve it to find the value of x.

Answer(a) $x =$ .... [3]

   (b) Find the integer values of t which satisfy the inequalities.

$$4t + 7 < 39 \leqslant 7t + 2$$

Answer(b) .... [3]

(c) Solve the following equations.

(i) $\dfrac{21 - x}{x + 3} = 4$

Answer(c)(i) $x =$ .... [3]

(ii) $3x^2 + 7x - 5 = 0$

Show all your working and give your answers correct to 2 decimal places.

Answer(c)(ii) $x =$ .... or $x =$ .... [4]

11 **(a)** Make x the subject of the formula.

$$A - x = \frac{xr}{t}$$

Answer(a) $x =$ .... [4]

(b) Find the value of a and the value of b when $x^2 - 16x + a = (x + b)^2$.

Answer(b) $a =$ ....

$b =$ .... [3]

(c) Write as a single fraction in its simplest form.

$$\frac{6}{x - 4} - \frac{5}{3x - 2}$$

Answer(c) .... [3]

May/June 2015 (42)

3 On the first part of a journey, Alan drove a distance of x km and his car used 6 litres of fuel.

The rate of fuel used by his car was $\frac{600}{x}$ litres per 100 km.

(a) Alan then drove another $(x + 20)$ km and his car used another 6 litres of fuel.

(i) Write down an expression, in terms of x, for the rate of fuel used by his car on this part of the journey.
Give your answer in litres per 100 km.

Answer(a)(i) .... litres per 100 km [1]

(ii) On this part of the journey the rate of fuel used by the car decreased by 1.5 litres per 100 km.

Show that $x^2 + 20x - 8000 = 0$.

Answer(a)(ii)

[4]

(b) Solve the equation $x^2 + 20x - 8000 = 0$.

Answer(b) x = ... or $x =$ .... [3]

(c) Find the rate of fuel used by Alan's car for the complete journey.
Give your answer in litres per 100 km.

Answer(c) .... litres per 100 km [2]

5      $y = x^2 - 2x + \dfrac{12}{x}$, $x \neq 0$

(a) Complete the table of values.

x	-4	-3	-2	-1	-0.5	0.5	1	2	3	4
y	21	11		-9	-22.75	23.25	11	6		11

[2]

(b) On the grid, draw the graph of $y = x^2 - 2x + \dfrac{12}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

[5]
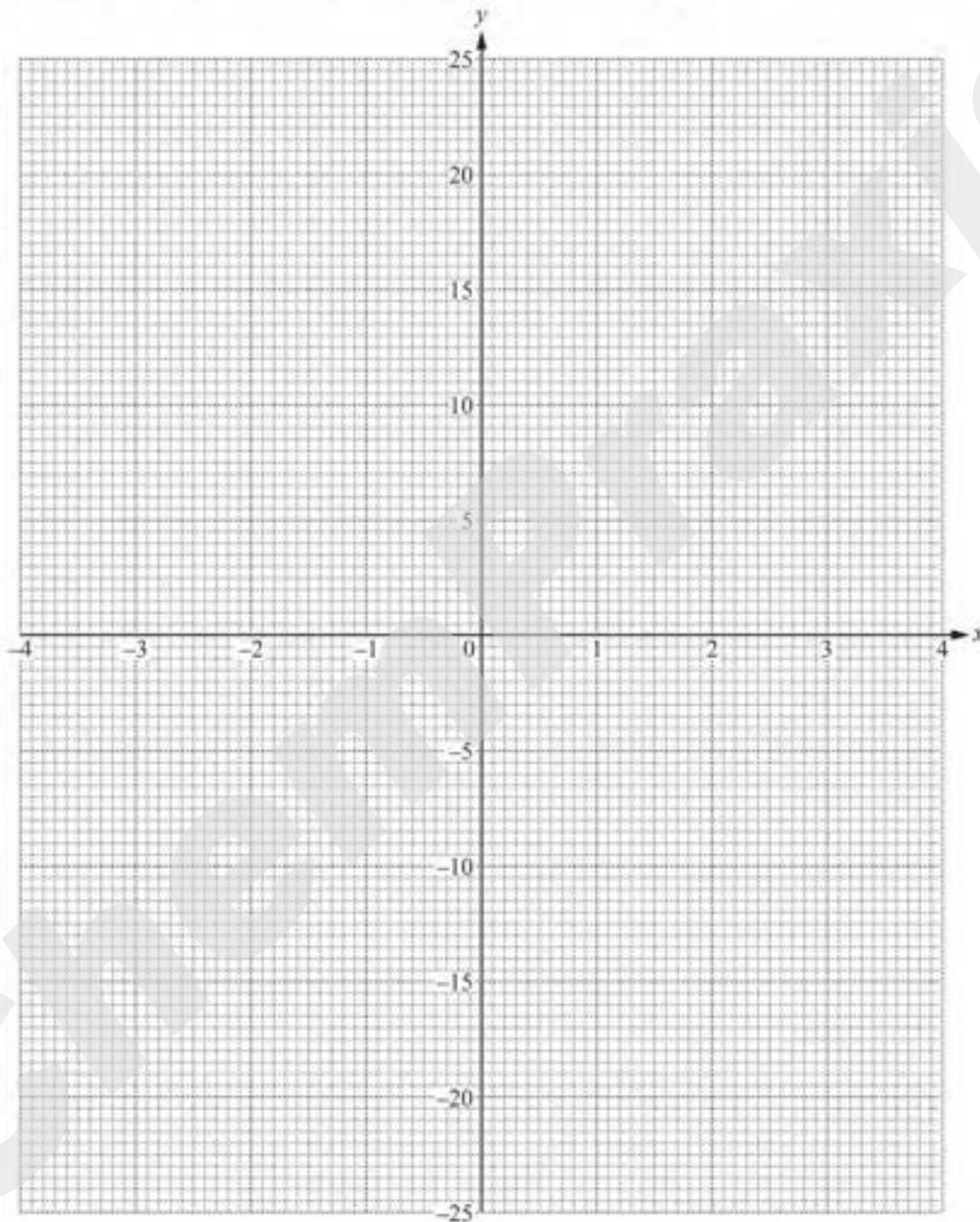
(c) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point (1, 11).

Answer(c) .... [3]

(d) The equation $x^2 - 2x + \dfrac{12}{x} = k$ has exactly two distinct solutions.

Use the graph to find

(i) the value of k,

Answer(d)(i) $k =$ .... [1]

(ii) the solutions of $x^2 - 2x + \dfrac{12}{x} = k$.

Answer(d)(ii) $x =$ .... or $x =$ .... [2]

(e) The equation $x^3 + ax^2 + bx + c = 0$ can be solved by drawing the line $y = 3x + 1$ on the grid.

Find the value of a, the value of b and the value of c.

Answer(e) $a =$ ....

$b =$ ....

$c =$ .... [3]

8   Sima sells x biscuits and y cakes.

   (a) (i)   She sells at least 100 biscuits.

             Write down an inequality in x.

             Answer(a)(i) .... [1]

       (ii) She sells at least 120 cakes.

             Write down an inequality in y.

             Answer(a)(ii) .... [1]

       (iii) She sells a maximum of 300 biscuits and cakes altogether.

             Write down an inequality in x and y.

             Answer(a)(iii) .... [1]

       (iv) Sima makes a profit of 40 cents on each biscuit and 80 cents on each cake.
             Her total profit is at least \$160.

             Show that $x + 2y \geq 400$.

             Answer(a)(iv)

             [1]

(b) On the grid, draw four lines to show the four inequalities and shade the unwanted regions.
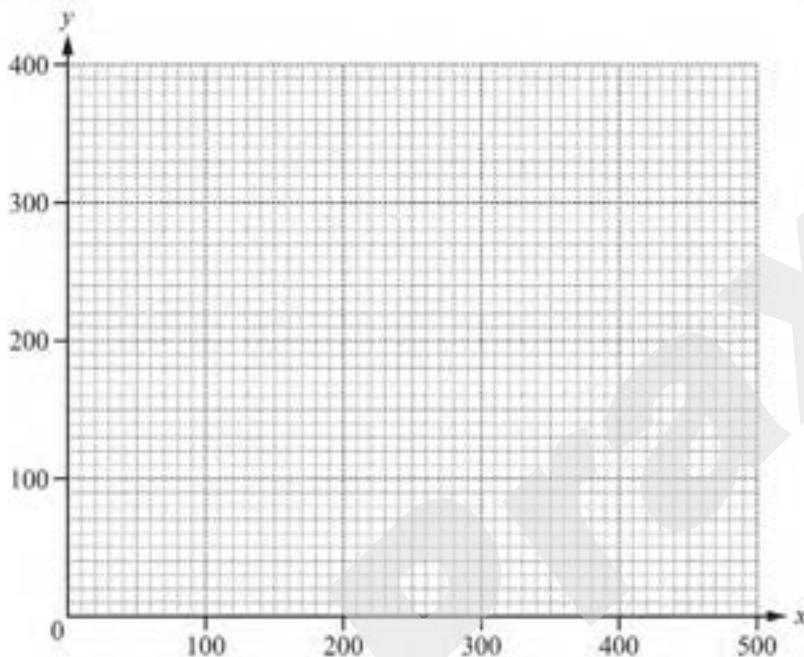
[6]

(c) Calculate Sima's maximum profit.
Give your answer in dollars.

Answer(c) $ .... [2]

9 (a) Expand and simplify.

$$3x(x-2) - 2x(3x-5)$$

Answer(a) .... [3]

(b) Factorise the following completely.

(i) $6w + 3wy - 4x - 2xy$

Answer(b)(i) .... [2]

(ii) $4x^2 - 25y^2$

Answer(b)(ii) .... [2]

(c) Simplify.

$$\left(\frac{16}{9x^4}\right)^{-\frac{1}{2}}$$

Answer(c) .... [2]

(d) n is an integer.

(i) Explain why $2n - 1$ is an odd number.

Answer(d)(i) ....

.... [1]

(ii) Write down, in terms of n, the next odd number after $2n - 1$.

Answer(d)(ii) .... [1]

(iii) Show that the difference between the squares of two consecutive odd numbers is a multiple of 8.

Answer(d)(iii)

[3]

May/June 2015 (43)

3      $f(x) = \dfrac{8}{x^2} + \dfrac{x}{2}$, $x \neq 0$.

(a) Complete the table of values for $f(x)$.

x	-5	-4	-3	-2	-1.5	1.5	2	2.5	3	3.5	4	5
$f(x)$	-2.2	-1.5	-0.6		2.8	4.3		2.5	2.4	2.4		2.8

[3]

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(b) On the grid, draw the graph of $y = f(x)$ for $-5 \leqslant x \leqslant -1.5$ and $1.5 \leqslant x \leqslant 5$.
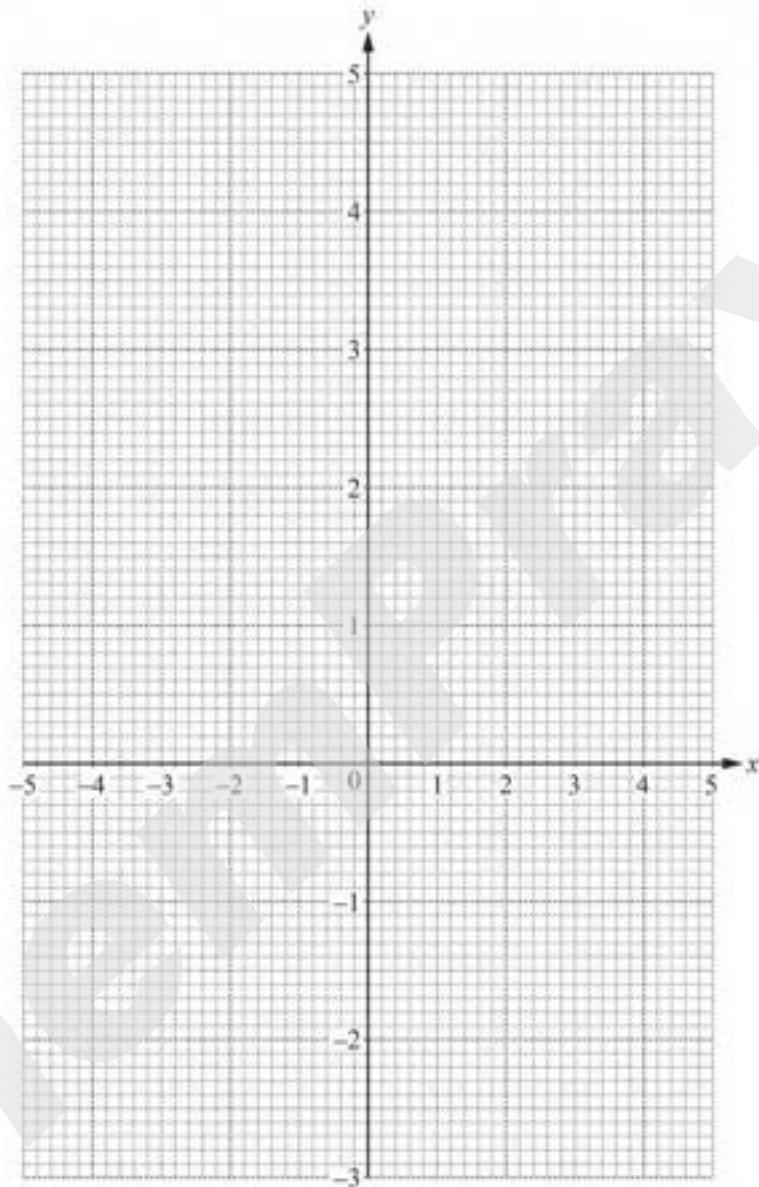
[5]

(c) Solve $f(x) = 0$.

Answer(c) x = ... [1]

(d) By drawing a suitable line on the grid, solve the equation $f(x) = 1 - x$.

Answer(d) x = ... [3]

(e) By drawing a tangent at the point $(-3, -0.6)$, estimate the gradient of the graph of $y = f(x)$ when $x = -3$.

Answer(e) .... [3]

7 (a) The total surface area of a cone is given by the formula $A = \pi r l + \pi r^2$.

(i) Find A when $r = 6.2$ cm and $l = 10.8$ cm.

Answer(a)(i) .... cm^2 [2]

(ii) Rearrange the formula to make l the subject.

Answer(a)(ii) l = ... [2]

(b) **(i)** Irina walks 10 km at 4 km/h and then a further 8 km at 5 km/h.

Calculate Irina's average speed for the whole journey.

Answer(b)(i) .... km/h [3]

(ii) Dariella walks x km at 5 km/h and then runs $(x + 4)$ km at 10 km/h.
The average speed of this journey is 7 km/h.

Find the value of x.
Show all your working.

Answer(b)(ii) $x =$ .... [5]

(c) (i) Priyantha sells her model car for $19.80 at a profit of 20%.

Calculate the original price of the model car.

Answer(c)(i) $.. [3]

(ii) Dev sells his model car for x at a profit of y%.

Find an expression, in terms of x and y, for the original price of this model car.
Write your answer as a single fraction.

Answer(c)(ii) $.. [3]

10    $f(x) = 2x - 1$ $g(x) = x^2 + x$ $h(x) = \dfrac{2}{x}, x \neq 0$

(a) Find ff(3).

Answer(a) .... [2]

(b) Find gf(x), giving your answer in its simplest form.

Answer(b) .... [3]

(c) Find $f^{-1}(x)$.

Answer(c) $f^{-1}(x) = $ .... [2]

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(d) Find $h(x) + h(x + 2)$, giving your answer as a single fraction.

Answer(d) .... [4]

11 The first four terms of sequences A, B, C and D are shown in the table.

Sequence	1st term	2nd term	3rd term	4th term	5th term	nth term
A	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$		
B	3	4	5	6		
C	-1	0	1	2		
D	-3	0	5	12		

(a) Complete the table.

[8]

(b) Which term in sequence A is equal to $\frac{36}{37}$?

Answer(b) .... [2]

(c) Which term in sequence D is equal to 725?

Answer(c) .... [2]

October/November 2015 (41)

2 **(a)** Calculate $2^{0.7}$.

Answer(a) .... [1]

(b) Find the value of x in each of the following.

(i) $2^x = 128$

Answer(b)(i) x = ... [1]

(ii) $2^x \times 2^9 = 2^{13}$

Answer(b)(ii) x = ... [1]

(iii) $2^9 \div 2^x = 4$

Answer(b)(iii) x = ... [1]

(iv) $2^x = \sqrt[3]{2}$

Answer(b)(iv) x = ... [1]

(c) (i) Complete this table of values for $y = 2^x$.

x	-3	-2	-1	0	1	2	3
y	0.125		0.5		2	4	8

[2]

(ii) On the grid, draw the graph of $y = 2^x$ for $-3 \leqslant x \leqslant 3$.
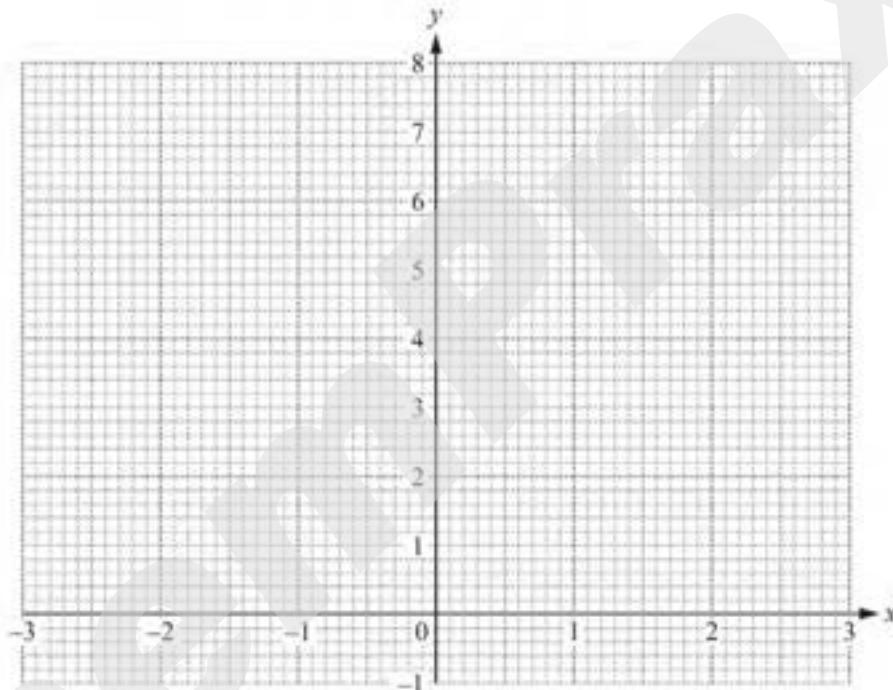
[4]

(iii) Use your graph to solve $2^x = 5$.

Answer(c)(iii) x = ... [1]

(iv) Find the equation of the line joining the points $(1, 2)$ and $(3, 8)$.

Answer(c)(iv) .... [3]

(v) By drawing a suitable line on your graph, solve $2^x - 2 - x = 0$.

Answer(c)(v) $x =$ .... or $x =$ .... [2]

4 Ali buys x rose bushes and y lavender bushes.

He buys:
- at least 5 rose bushes
- at most 8 lavender bushes
- at most 15 bushes in total
- more lavender bushes than rose bushes.

(a) (i) Write down four inequalities, in terms of x and/or y, to show this information.

Answer(a)(i) ....

....

...

.... [4]

(ii) On the grid, show the information in **part (a)(i)** by drawing four straight lines.
Label the region R where all four inequalities are true.
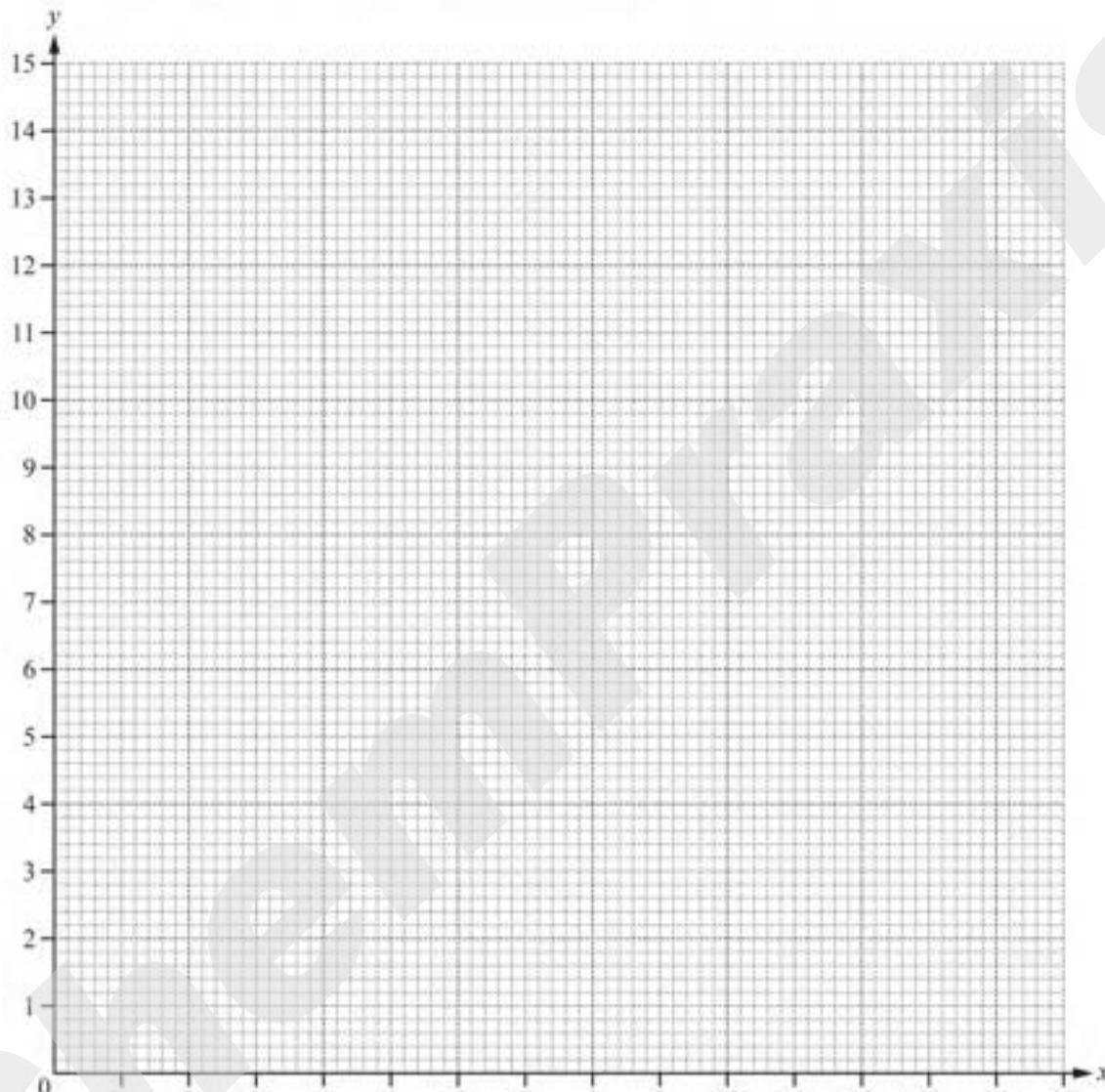
[5]

(b) Rose bushes cost \$6 each and lavender bushes cost \$4.50 each.

What is the greatest amount of money Ali could spend?

Answer(b) \$ .... [2]

8 **(a)** Factorise $x^2 - 3x - 10$.

Answer(a) .... [2]

(b) (i) Show that $\dfrac{x+2}{x+1} + \dfrac{3}{x} = 3$ simplifies to $2x^2 - 2x - 3 = 0$.

Answer(b)(i)

[3]

(ii) Solve $2x^2 - 2x - 3 = 0$.
Give your answers correct to 3 decimal places.
Show all your working.

Answer(b)(ii) $x =$ .... or $x =$ .... [4]

(c) Simplify $\dfrac{2x+3}{x+2} - \dfrac{x}{x+1}$.

Answer(c) .... [4]

9 The first three diagrams in a sequence are shown below.
   The diagrams are made by drawing lines of length 1 cm.
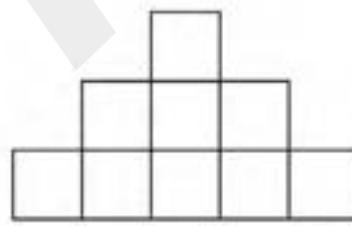
Diagram 1   Diagram 2   Diagram 3

(a) The areas of each of the first three diagrams are shown in this table.

Diagram	1	2	3
Area (cm²)	1	4	9

(i) Find the area of Diagram 4.

Answer(a)(i) .... cm² [1]

(ii) Find, in terms of n, the area of Diagram n.

Answer(a)(ii) .... cm² [1]

(b) The numbers of 1 cm lines needed to draw each of the first three diagrams are shown in this table.

Diagram	1	2	3
Number of 1 cm lines	4	13	26

(i) Find the number of 1 cm lines needed to draw Diagram 4.

Answer(b)(i) .... [1]

(ii) In which diagram are 118 lines of length 1 cm needed?

Answer(b)(ii) .... [1]

(c) The **total** number of 1 cm lines needed to draw both Diagram 1 and Diagram 2 is 17.
The **total** number of 1 cm lines needed to draw all of the first n diagrams is

$$\frac{2}{3}n^3 + an^2 + bn.$$

Find the value of a and the value of b.
Show all your working.

Answer(c) a = ...

b = ... [6]

October/November 2015 (42)

2 The table shows some values for $y = x^3 - 3x + 2$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y		3.125		3.375	2		0		4

(a) Complete the table of values. [4]

(b) On the grid, draw the graph of $y = x^3 - 3x + 2$ for $-2 \leqslant x \leqslant 2$.

[4]
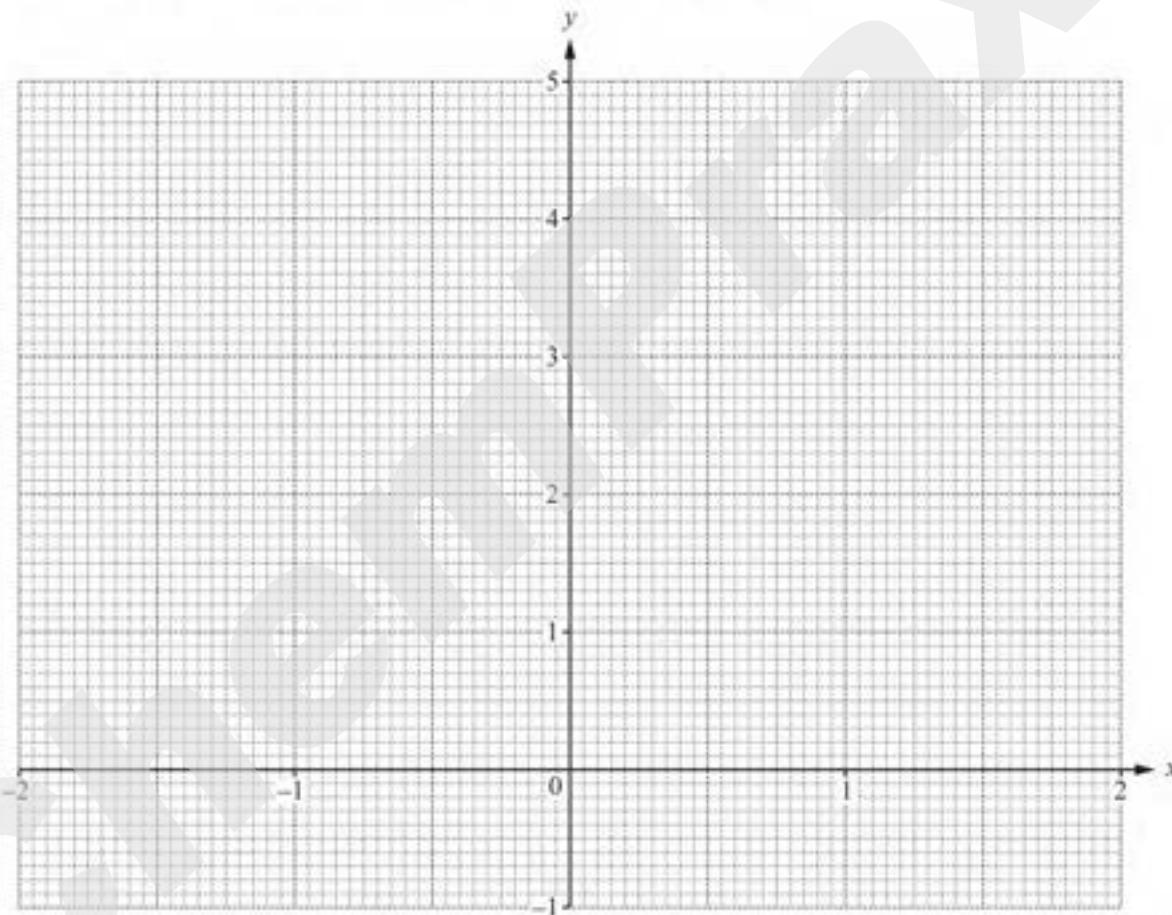
(c) By drawing a suitable line, solve the equation $x^3 - 3x + 2 = x + 1$ for $-2 \leqslant x \leqslant 2$.

Answer(c) $x =$ .... or $x =$ .... [3]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.5$.

Answer(d) .... [3]

Question 5b and 5c

(b) Solve the simultaneous equations.
You must show all your working.

$$3x - 2y = 23$$
$$-4x - y = -5$$

Answer(b) $x =$ ....

$y =$ .... [3]

(c) Solve the equation.

$$\frac{2(t+3)}{t} - \frac{t}{t+3} = 1$$

Answer(c) $t =$ .... [5]

9         $f(x) = 2x + 5$ $g(x) = 2^x$ $h(x) = 7 - 3x$

(a)   Find

    (i) $f(3)$,

Answer(a)(i) .... [1]

    (ii) $gg(3)$.

Answer(a)(ii) .... [2]

(b)   Find $f^{-1}(x)$.

Answer(b) $f^{-1}(x) =$ .... [2]

(c)   Find $fh(x)$, giving your answer in its simplest form.

Answer(c) .... [2]

(d) Find the integer values of x which satisfy this inequality.

$$1 < f(x) \leq 9$$

Answer(d) .... [3]

10   The table shows the first five terms of sequences A, B and C.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term
A	3	4	5	6	7	
B	0	1	4	9	16	
C	−3	−3	−1	3	9	

(a) Complete the table for the 6th term of each sequence.   [2]

(b) Write down the nth term of sequence A.

Answer(b) .... [1]

(c) **(i)** Find the nth term of sequence B.

Answer(c)(i) .... [2]

(ii) Find the value of n when the nth term of sequence B is 8281.

Answer(c)(ii) $n =$ .... [2]

(d) **(i)** Find the nth term of sequence C in its simplest form.

Answer(d)(i) .... [2]

(ii) Find the 8th term of sequence C.

Answer(d)(ii) .... [1]

(e) The nth term of another sequence D is $\left(-\dfrac{1}{2}\right)^{n-1}$.

Complete the table for the first four terms of sequence D.

| Sequence	1st term	2nd term	3rd term	4th term
D				

[3]

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4        $f(x) = x - \dfrac{1}{2x^2}, \quad x \neq 0$

(a) Complete the table of values.

x	-3	-2	-1.5	-1	-0.5	-0.3		0.3	0.5	1	1.5	2
$f(x)$	-3.1	-2.1	-1.7		-2.5	-5.9		-5.3	-1.5		1.3	1.9

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 2$.

[5]
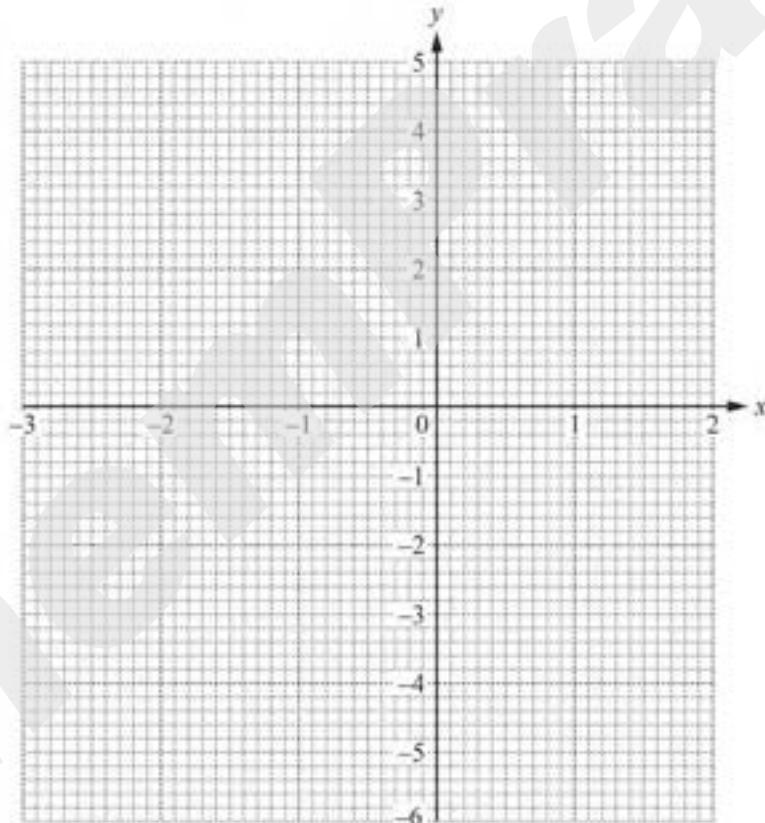
(c) Use your graph to solve the equation $f(x) = 1$.

Answer(c) x = ... [1]

(d) There is only one negative integer value, k, for which $f(x) = k$ has only one solution for all real x.

Write down this value of k.

Answer(d) k = ... [1]

(e) The equation $2x - \dfrac{1}{2x^2} - 2 = 0$ can be solved using the graph of $y = f(x)$ and a straight line graph.

(i) Find the equation of this straight line.

Answer(e)(i) y = ... [1]

(ii) On the grid, draw this straight line and solve the equation $2x - \dfrac{1}{2x^2} - 2 = 0$.

Answer(e)(ii) x = ... [3]

7   **(a)** The cost of a loaf of bread is x cents.
The cost of a cake is $(x - 5)$ cents.
The total cost of 6 loaves of bread and 11 cakes is \$13.56 .

Find the value of x.

Answer(a) x = ... [4]

(c) The cost of a bottle of water is $(w - 1)$ cents.
The cost of a bottle of milk is $(2w - 11)$ cents.
A certain number of bottles of water costs $4.80 .
The same number of bottles of milk costs $7.80 .

Find the value of w.

Answer(c) w = ... [4]

9        $f(x) = 2x - 1$ $g(x) = \frac{1}{x}, \; x \neq 0$ $h(x) = 2^x$

(a) Find $h(3)$.

Answer(a) .... [1]

(b) Find $fg(0.5)$.

Answer(b) .... [2]

(c) Find $f^{-1}(x)$.

Answer(c) f^{-1}(x) = ... [2]

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(d) Find ff(x), giving your answer in its simplest form.

Answer(d) .... **[2]**

(e) Find $(f(x))^2 + 6$, giving your answer in its simplest form.

Answer(e) .... **[2]**

(f) Simplify hh^{-1}(x).

Answer(f) .... **[1]**

(g) Which of the following statements is true?

$f^{-1}(x) = f(x)$

$g^{-1}(x) = g(x)$

$h^{-1}(x) = h(x)$

Answer(g) .... **[1]**

(h) Use two of the functions $f(x)$, $g(x)$ and $h(x)$ to find the composite function which is equal to $2^{x+1} - 1$.

Answer(h) .... [1]

10   Complete the table for each sequence.

Sequence	1st term	2nd term	3rd term	4th term	5th term	6th term		nth term
A	15	8	1	−6				
B	$\frac{5}{18}$	$\frac{6}{19}$	$\frac{7}{20}$	$\frac{8}{21}$				
C	2	5	10	17				
D	2	6	18	54				

[11]