Factors of polynomials

(Past Year Topical Questions 2012-2017)

Oct/Nov 2012 (12)

10.

A function f is such that $f(x) = 4x^3 + 4x^2 + ax + b$. It is given that $2x - 1$ is a factor of both $f(x)$ and $f'(x)$.

(i) Show that $b = 2$ and find the value of a. [5]

Using the values of a and b from part **(i)**,

(ii) find the remainder when $f(x)$ is divided by $x + 3$, [2]

(iii) express $f(x)$ in the form $f(x) = (2x - 1)(px^2 + qx + r)$, where p, q and r are integers to be found, [2]

(iv) find the values of x for which $f(x) = 0$. [2]

May/June 2013 (12)

7.

It is given that $f(x) = 6x^3 - 5x^2 + ax + b$ has a factor of $x + 2$ and leaves a remainder of 27 when divided by $x - 1$.

(i) Show that $b = 40$ and find the value of a. [4]

(ii) Show that $f(x) = (x + 2)(px^2 + qx + r)$, where p, q and r are integers to be found. [2]

(iii) Hence solve $f(x) = 0$. [2]

Oct/Nov 2013 (11)

6.

The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is such that $2x - 1$ is a factor. Given that the remainder when $f(x)$ is divided by $x - 2$ is twice the remainder when $f(x)$ is divided by $x + 1$, find the value of a and of b. [6]

May/June 2014 (13)

3.

(i) Find, in terms of p, the remainder when $x^3 + px^2 + p^2x + 21$ is divided by $x + 3$. [2]

(ii) Hence find the set of values of p for which this remainder is negative. [3]

May/June 2015 (13)

6.

The polynomial $f(x) = ax^3 - 15x^2 + bx - 2$ has a factor of $2x - 1$ and a remainder of 5 when divided by $x - 1$.

(i) Show that $b = 8$ and find the value of a. [4]

(ii) Using the values of a and b from part (i), express $f(x)$ in the form $(2x - 1)\,g(x)$, where $g(x)$ is a quadratic factor to be found. [2]

(iii) Show that the equation $f(x) = 0$ has only one real root. [2]

May/June 2016 (11)

10.

(i) Given that $f(x) = 4x^3 + kx + p$ is exactly divisible by $x + 2$ and $f'(x)$ is exactly divisible by $2x - 1$, find the value of k and of p. [4]

(ii) Using the values of k and p found in part **(i)**, show that $f(x) = (x + 2)(ax^2 + bx + c)$, where a, b and c are integers to be found. [2]

(iii) Hence show that $f(x) = 0$ has only one solution and state this solution. [2]

Oct/Nov 2016 (11)

9.

The polynomial $p(x)$ is $ax^3 - 4x^2 + bx + 18$. It is given that $p(x)$ and $p'(x)$ are both divisible by $2x - 3$.

(i) Show that $a = 4$ and find the value of b. [4]

(ii) Using the values of a and b from part **(i)**, factorise $p(x)$ completely. [2]

(iii) Hence find the values of x for which $p(x) = x + 2$. [3]

May/June 2017 (11)

2.

It is given that $p(x) = x^3 + ax^2 + bx - 48$. When $p(x)$ is divided by $x - 3$ the remainder is 6. Given that $p'(1) = 0$, find the value of a and of b. [5]

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8.

It is given that $p(x) = 2x^3 + ax^2 + 4x + b$, where a and b are constants. It is given also that $2x + 1$ is a factor of $p(x)$ and that when $p(x)$ is divided by $x - 1$ there is a remainder of -12.

(i) Find the value of a and of b. [5]

(ii) Using your values of a and b, write $p(x)$ in the form $(2x + 1)q(x)$, where $q(x)$ is a quadratic expression. [2]

(iii) Hence find the exact solutions of the equation $p(x) = 0$. [2]

Oct/Nov 2017 (11)

2.

The polynomial $p(x)$ is $ax^3 + bx^2 - 13x + 4$, where a and b are integers. Given that $2x - 1$ is a factor of $p(x)$ and also a factor of $p'(x)$,

(i) find the value of a and of b. [5]

Using your values of a and b,

(ii) find the remainder when $p(x)$ is divided by $x + 1$. [2]

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7.

A polynomial $p(x)$ is $ax^3 + 8x^2 + bx + 5$, where a and b are integers. It is given that $2x - 1$ is a factor of $p(x)$ and that a remainder of -25 is obtained when $p(x)$ is divided by $x + 2$.

(i) Find the value of a and of b. [5]

(ii) Using your values of a and b, find the exact solutions of $p(x) = 5$. [2]