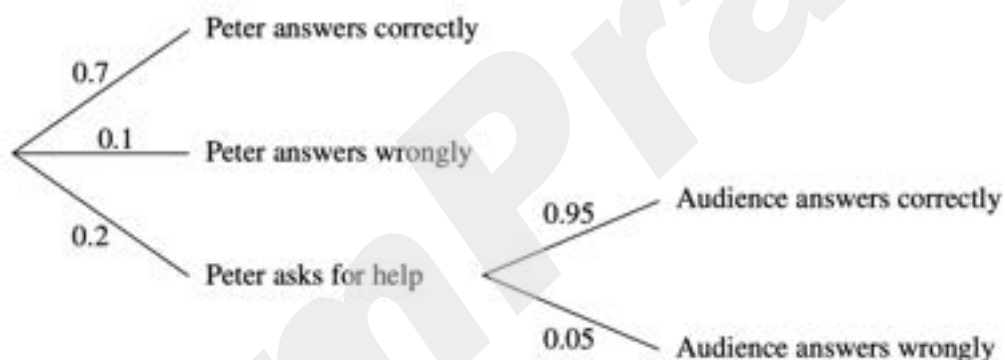# Probability
(Past Year Topical Questions 2010-2015)

May/June 2010 (61)

7   In a television quiz show Peter answers questions one after another, stopping as soon as a question is answered wrongly.

- The probability that Peter gives the correct answer himself to any question is 0.7.
- The probability that Peter gives a wrong answer himself to any question is 0.1.
- The probability that Peter decides to ask for help for any question is 0.2.

On the first occasion that Peter decides to ask for help he asks the audience. The probability that the audience gives the correct answer to any question is 0.95. This information is shown in the tree diagram below.

(i) Show that the probability that the first question is answered correctly is 0.89.   [1]

On the second occasion that Peter decides to ask for help he phones a friend. The probability that his friend gives the correct answer to any question is 0.65.

(ii) Find the probability that the first two questions are both answered correctly.   [6]

(iii) Given that the first two questions were both answered correctly, find the probability that Peter asked the audience.   [3]

<u>May/June 2010 (62)</u>

5 Two fair twelve-sided dice with sides marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are thrown, and the numbers on the sides which land face down are noted. Events Q and R are defined as follows.

Q : the product of the two numbers is 24.

R : both of the numbers are greater than 8.

(i) Find P(Q). [2]

(ii) Find P(R). [2]

(iii) Are events Q and R exclusive? Justify your answer. [2]

(iv) Are events Q and R independent? Justify your answer. [2]

6 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

<u>May/June 2010 (63)</u>

1 A bottle of sweets contains 13 red sweets, 13 blue sweets, 13 green sweets and 13 yellow sweets. 7 sweets are selected at random. Find the probability that exactly 3 of them are red. [3]

5 Three friends, Rick, Brenda and Ali, go to a football match but forget to say which entrance to the ground they will meet at. There are four entrances, A, B, C and D. Each friend chooses an entrance independently.

- The probability that Rick chooses entrance A is $\frac{1}{3}$. The probabilities that he chooses entrances B, C or D are all equal.

- Brenda is equally likely to choose any of the four entrances.

- The probability that Ali chooses entrance C is $\frac{2}{9}$ and the probability that he chooses entrance D is $\frac{1}{3}$. The probabilities that he chooses the other two entrances are equal.

(i) Find the probability that at least 2 friends will choose entrance B. [4]

(ii) Find the probability that the three friends will all choose the same entrance. [4]

3 A fair five-sided spinner has sides numbered 1, 2, 3, 4, 5. Raj spins the spinner and throws two fair dice. He calculates his score as follows.

- If the spinner lands on an **even**-numbered side, Raj **multiplies** the two numbers showing on the dice to get his score.

- If the spinner lands on an odd-numbered side, Raj **adds** the numbers showing on the dice to get his score.

Given that Raj's score is 12, find the probability that the spinner landed on an even-numbered side.

[6]

3 It was found that 68% of the passengers on a train used a cell phone during their train journey. Of those using a cell phone, 70% were under 30 years old, 25% were between 30 and 65 years old and the rest were over 65 years old. Of those not using a cell phone, 26% were under 30 years old and 64% were over 65 years old.

(i) Draw a tree diagram to represent this information, giving all probabilities as decimals. [2]

(ii) Given that one of the passengers is 45 years old, find the probability of this passenger using a cell phone during the journey. [3]

2 When Ted is looking for his pen, the probability that it is in his pencil case is 0.7. If his pen is in his pencil case he always finds it. If his pen is somewhere else, the probability that he finds it is 0.2. Given that Ted finds his pen when he is looking for it, find the probability that it was in his pencil case. [4]

Question 7b

(b) A bag contains 5 green balls and 3 yellow balls. Ronnie and Julie play a game in which they take turns to draw a ball from the bag at random without replacement. The winner of the game is the first person to draw a yellow ball. Julie draws the first ball. Find the probability that Ronnie wins the game. [4]

4 Tim throws a fair die twice and notes the number on each throw.

(i) Tim calculates his final score as follows. If the number on the second throw is a 5 he multiplies the two numbers together, and if the number on the second throw is not a 5 he adds the two numbers together. Find the probability that his final score is

    (a) 12, [1]

    (b) 5. [3]

(ii) Events A, B, C are defined as follows.

    A: the number on the second throw is 5

    B: the sum of the numbers is 6

    C: the product of the numbers is even

By calculation find which pairs, if any, of the events A, B and C are independent. [5]

7   Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

    (i) Find the probability that exactly two of the selected balls have the same number.   [5]

    (ii) Given that exactly two of the selected balls have the same number, find the probability that they are both numbered 2.   [2]

    (iii) Event X is 'exactly two of the selected balls have the same number'. Event Y is 'the ball selected from bag A has number 2'. Showing your working, determine whether events X and Y are independent or not.   [2]

5   A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

    (i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$.   [1]

    (ii) The spinner is spun 3 times. Find the probability that it lands on a different coloured side each time.   [3]

2   In a group of 30 teenagers, 13 of the 18 males watch 'Kops are Kids' on television and 3 of the 12 females watch 'Kops are Kids'.

    (i) Find the probability that a person chosen at random from the group is either female or watches 'Kops are Kids' or both.   [4]

    (ii) Showing your working, determine whether the events 'the person chosen is male' and 'the person chosen watches Kops are Kids' are independent or not.   [2]

May/June 2012 (61)

2 Maria has 3 pre-set stations on her radio. When she switches her radio on, there is a probability of 0.3 that it will be set to station 1, a probability of 0.45 that it will be set to station 2 and a probability of 0.25 that it will be set to station 3. On station 1 the probability that the presenter is male is 0.1, on station 2 the probability that the presenter is male is 0.85 and on station 3 the probability that the presenter is male is p. When Maria switches on the radio, the probability that it is set to station 3 and the presenter is male is 0.075.

   (i) Show that the value of p is 0.3. [1]

   (ii) Given that Maria switches on and hears a male presenter, find the probability that the radio was set to station 2. [4]

May/June 2012 (62)

6 A box of biscuits contains 30 biscuits, some of which are wrapped in gold foil and some of which are unwrapped. Some of the biscuits are chocolate-covered. 12 biscuits are wrapped in gold foil, and of these biscuits, 7 are chocolate-covered. There are 17 chocolate-covered biscuits in total.

   (i) Copy and complete the table below to show the number of biscuits in each category. [2]

	Wrapped in gold foil	Unwrapped	Total
Chocolate-covered			
Not chocolate-covered			
Total			30

A biscuit is selected at random from the box.

   (ii) Find the probability that the biscuit is wrapped in gold foil. [1]

The biscuit is returned to the box. An unwrapped biscuit is then selected at random from the box.

   (iii) Find the probability that the biscuit is chocolate-covered. [1]

The biscuit is returned to the box. A biscuit is then selected at random from the box.

   (iv) Find the probability that the biscuit is unwrapped, given that it is chocolate-covered. [1]

The biscuit is returned to the box. Nasir then takes 4 biscuits without replacement from the box.

   (v) Find the probability that he takes exactly 2 wrapped biscuits. [4]

May/June 2012 (63)

5 Suzanne has 20 pairs of shoes, some of which have designer labels. She has 6 pairs of high-heeled shoes, of which 2 pairs have designer labels. She has 4 pairs of low-heeled shoes, of which 1 pair has designer labels. The rest of her shoes are pairs of sports shoes. Suzanne has 8 pairs of shoes with designer labels in total.

(i) Copy and complete the table below to show the number of pairs in each category. [2]

	Designer labels	No designer labels	Total
High-heeled shoes			
Low-heeled shoes			
Sports shoes			
Total			20

Suzanne chooses 1 pair of shoes at random to wear.

(ii) Find the probability that she wears the pair of low-heeled shoes with designer labels. [1]

(iii) Find the probability that she wears a pair of sports shoes. [1]

(iv) Find the probability that she wears a pair of high-heeled shoes, given that she wears a pair of shoes with designer labels. [1]

(v) State with a reason whether the events 'Suzanne wears a pair of shoes with designer labels' and 'Suzanne wears a pair of sports shoes' are independent. [2]

Suzanne chooses 1 pair of shoes at random each day.

(vi) Find the probability that Suzanne wears a pair of shoes with designer labels on at most 4 days out of the next 7 days. [3]

<u>October/November 2012 (61)</u>

4 Prices in dollars of 11 caravans in a showroom are as follows.

16 800   18 500   17 700   14 300   15 500   15 300   16 100   16 800   17 300   15 400   16 400

   (i) Represent these prices by a stem-and-leaf diagram. [3]

   (ii) Write down the lower quartile of the prices of the caravans in the showroom. [1]

   (iii) 3 different caravans in the showroom are chosen at random and their prices are noted. Find the probability that 2 of these prices are more than the median and 1 is less than the lower quartile. [3]

<u>October/November 2012 (62)</u>

1 Fabio drinks coffee each morning. He chooses Americano, Cappucino or Latte with probabilities 0.5, 0.3 and 0.2 respectively. If he chooses Americano he either drinks it immediately with probability 0.8, or leaves it to drink later. If he chooses Cappucino he either drinks it immediately with probability 0.6, or leaves it to drink later. If he chooses Latte he either drinks it immediately with probability 0.1, or leaves it to drink later.

   (i) Find the probability that Fabio chooses Americano and leaves it to drink later. [1]

   (ii) Fabio drinks his coffee immediately. Find the probability that he chose Latte. [4]

3 Ronnie obtained data about the gross domestic product (GDP) and the birth rate for 170 countries. He classified each GDP and each birth rate as either 'low', 'medium' or 'high'. The table shows the number of countries in each category.

		Birth rate		
		Low	Medium	High
GDP	Low	3	5	45
	Medium	20	42	12
	High	35	8	0

One of these countries is chosen at random.

(i) Find the probability that the country chosen has a medium GDP. [1]

(ii) Find the probability that the country chosen has a low birth rate, given that it does not have a medium GDP. [2]

(iii) State with a reason whether or not the events 'the country chosen has a high GDP' and 'the country chosen has a high birth rate' are exclusive. [2]

One country is chosen at random from those countries which have a medium GDP and then a different country is chosen at random from those which have a medium birth rate.

(iv) Find the probability that both countries chosen have a medium GDP and a medium birth rate. [3]

7   Box A contains 8 white balls and 2 yellow balls. Box B contains 5 white balls and x yellow balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(i)   Justify the probability $\frac{x}{x+6}$ on the tree diagram.   [1]

(ii) Copy and complete the tree diagram.   [4]

(iii) If the ball chosen from box A is white then the probability that the ball chosen from box B is also white is $\frac{1}{3}$. Show that the value of x is 12.   [2]

(iv) Given that the ball chosen from box B is yellow, find the conditional probability that the ball chosen from box A was yellow.   [4]

7   Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

(i)   Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag.   [2]

(ii) Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate.   [4]

May/June 2013 (63)

1   Q is the event 'Nicola throws two fair dice and gets a total of 5'. S is the event 'Nicola throws two fair dice and gets one low score (1, 2 or 3) and one high score (4, 5 or 6)'. Are events Q and S independent? Justify your answer.   [4]

5   (a)   John plays two games of squash. The probability that he wins his first game is 0.3. If he wins his first game, the probability that he wins his second game is 0.6. If he loses his first game, the probability that he wins his second game is 0.15. Given that he wins his second game, find the probability that he won his first game.   [4]

   (b)   Jack has a pack of 15 cards. 10 cards have a picture of a robot on them and 5 cards have a picture of an aeroplane on them. Emma has a pack of cards. 7 cards have a picture of a robot on them and $x - 3$ cards have a picture of an aeroplane on them. One card is taken at random from Jack's pack and one card is taken at random from Emma's pack. The probability that both cards have pictures of robots on them is $\frac{7}{18}$. Write down an equation in terms of x and hence find the value of x.   [4]

October/November 2013 (61)

2   The people living in two towns, Mumbok and Bagville, are classified by age. The numbers in thousands living in each town are shown in the table below.

	Mumbok	Bagville
Under 18 years	15	35
18 to 60 years	55	95
Over 60 years	20	30

One of the towns is chosen. The probability of choosing Mumbok is 0.6 and the probability of choosing Bagville is 0.4. Then a person is chosen at random from that town. Given that the person chosen is between 18 and 60 years old, find the probability that the town chosen was Mumbok.   [5]

2 On Saturday afternoons Mohit goes shopping with probability 0.25, or goes to the cinema with probability 0.35 or stays at home. If he goes shopping the probability that he spends more than $50 is 0.7. If he goes to the cinema the probability that he spends more than $50 is 0.8. If he stays at home he spends $10 on a pizza.

    (i) Find the probability that Mohit will go to the cinema and spend less than $50.   [1]

    (ii) Given that he spends less than $50, find the probability that he went to the cinema.   [4]

5 Playground equipment consists of swings (S), roundabouts (R), climbing frames (C) and play-houses (P). The numbers of pieces of equipment in each of 3 playgrounds are as follows.

Playground X	Playground Y	Playground Z
3S, 2R, 4P	6S, 3R, 1C, 2P	8S, 3R, 4C, 1P

Each day Nur takes her child to one of the playgrounds. The probability that she chooses playground X is $\frac{1}{4}$. The probability that she chooses playground Y is $\frac{1}{4}$. The probability that she chooses playground Z is $\frac{1}{2}$. When she arrives at the playground, she chooses one piece of equipment at random.

    (i) Find the probability that Nur chooses a play-house.   [4]

    (ii) Given that Nur chooses a climbing frame, find the probability that she chose playground Y.   [4]

3 Roger and Andy play a tennis match in which the first person to win two sets wins the match. The probability that Roger wins the first set is 0.6. For sets after the first, the probability that Roger wins the set is 0.7 if he won the previous set, and is 0.25 if he lost the previous set. No set is drawn.

    (i) Find the probability that there is a winner of the match after exactly two sets.   [3]

    (ii) Find the probability that Andy wins the match given that there is a winner of the match after exactly two sets.   [2]

6 Tom and Ben play a game repeatedly. The probability that Tom wins any game is 0.3. Each game is won by either Tom or Ben. Tom and Ben stop playing when one of them (to be called the champion) has won two games.

    (i) Find the probability that Ben becomes the champion after playing exactly 2 games.   [1]

    (ii) Find the probability that Ben becomes the champion.   [3]

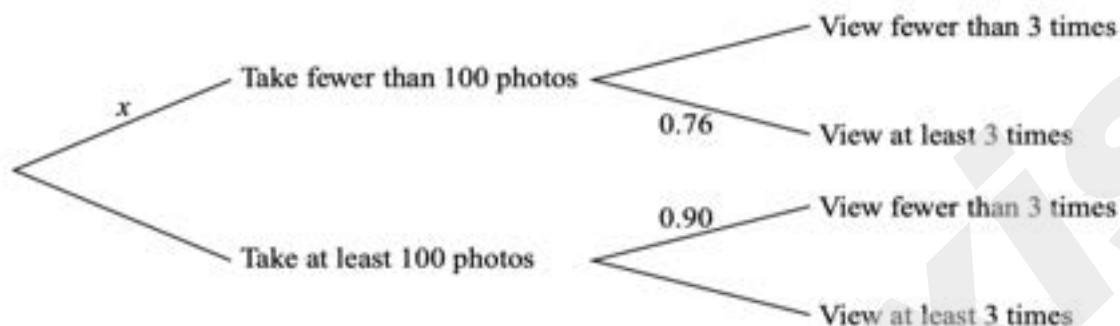   **(iii)** Given that Tom becomes the champion, find the probability that he won the 2nd game.   [4]

3 Jodie tosses a biased coin and throws two fair tetrahedral dice. The probability that the coin shows a head is $\frac{1}{3}$. Each of the dice has four faces, numbered 1, 2, 3 and 4. Jodie's score is calculated from the numbers on the faces that the dice land on, as follows:

    • if the coin shows a head, the two numbers from the dice are added together;

    • if the coin shows a tail, the two numbers from the dice are multiplied together.

Find the probability that the coin shows a head given that Jodie's score is 8.   [5]

3 Jason throws two fair dice, each with faces numbered 1 to 6. Event A is 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B is 'the product of the two numbers obtained is even'.

    (i) Determine whether events A and B are independent, showing your working.   [5]

    (ii) Are events A and B mutually exclusive? Justify your answer.   [1]

4

```
                                                          ── View fewer than 3 times
              Take fewer than 100 photos ──┤
      x  ─┤                            0.76  ── View at least 3 times
          │
          │                            0.90  ── View fewer than 3 times
              Take at least 100 photos ──┤
                                              ── View at least 3 times
```

A survey is undertaken to investigate how many photos people take on a one-week holiday and also how many times they view past photos. For a randomly chosen person, the probability of taking fewer than 100 photos is x. The probability that these people view past photos at least 3 times is 0.76. For those who take at least 100 photos, the probability that they view past photos fewer than 3 times is 0.90. This information is shown in the tree diagram. The probability that a randomly chosen person views past photos fewer than 3 times is 0.801.

(i) Find x. [3]

(ii) Given that a person views past photos at least 3 times, find the probability that this person takes at least 100 photos. [4]

May/June 2015 (62)

4
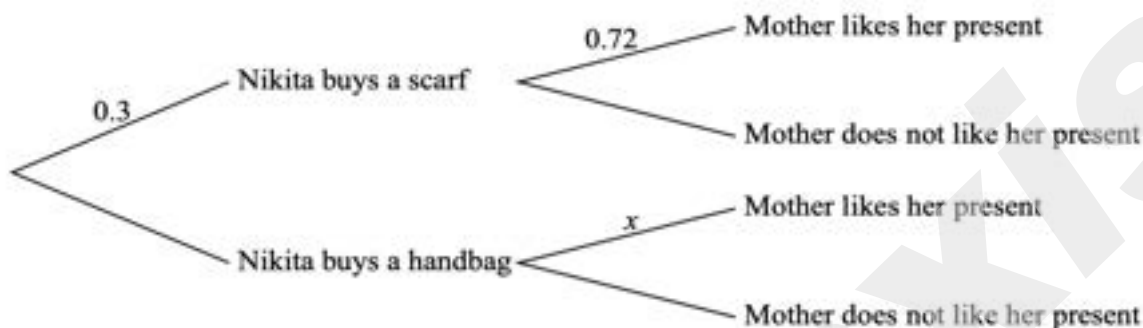
Nikita goes shopping to buy a birthday present for her mother. She buys either a scarf, with probability 0.3, or a handbag. The probability that her mother will like the choice of scarf is 0.72. The probability that her mother will like the choice of handbag is x. This information is shown on the tree diagram. The probability that Nikita's mother likes the present that Nikita buys is 0.783.

(i) Find x. [3]

(ii) Given that Nikita's mother does not like her present, find the probability that the present is a scarf. [4]

5   A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number.

2 When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

2 A committee of 6 people is to be chosen at random from 7 men and 9 women. Find the probability that there are no men on the committee. [3]

3 One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

October/November 2015 (63)

2   In country X, 25% of people have fair hair. In country Y, 60% of people have fair hair. There are 20 million people in country X and 8 million people in country Y. A person is chosen at random from these 28 million people.

    (i) Find the probability that the person chosen is from country X. [1]

    (ii) Find the probability that the person chosen has fair hair. [2]

    (iii) Find the probability that the person chosen is from country X, given that the person has fair hair. [2]

3   Ellie throws two fair tetrahedral dice, each with faces numbered 1, 2, 3 and 4. She notes the numbers on the faces that the dice land on. Event S is 'the sum of the two numbers is 4'. Event T is 'the product of the two numbers is an odd number'.

    (i) Determine whether events S and T are independent, showing your working. [5]

    (ii) Are events S and T exclusive? Justify your answer. [1]