Discrete Random Variables

(Past Year Topical Questions 2010-2015)

May/June 2010 (61)

1 The probability distribution of the discrete random variable X is shown in the table below.

x	-3	-1	0	4
$P(X = x)$	a	b	0.15	0.4

Given that $E(X) = 0.75$, find the values of a and b. [4]

4 The numbers of rides taken by two students, Fei and Graeme, at a fairground are shown in the following table.

	Roller coaster	Water slide	Revolving drum
Fei	4	2	0
Graeme	1	3	6

(i) The mean cost of Fei's rides is \$2.50 and the standard deviation of the costs of Fei's rides is \$0. Explain how you can tell that the roller coaster and the water slide each cost \$2.50 per ride. [2]

(ii) The mean cost of Graeme's rides is \$3.76. Find the standard deviation of the costs of Graeme's rides. [5]

May/June 2010 (62)

6 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

(ii) Show that $E(X) = \frac{8}{7}$ and calculate $\text{Var}(X)$. [3]

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3 Christa takes her dog for a walk every day. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is a probability of 0.75 that the dog will bark.

(i) Find the probability that they go to the park on more than 5 of the next 7 days. [2]

(ii) Find the probability that the dog barks on any particular day. [2]

(iii) Find the variance of the number of times they go to the park in 30 days. [1]

5 Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits.

(i) Show that $\text{P}(X = 2) = \frac{3}{7}$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $\text{E}(X)$ and $\text{Var}(X)$. [3]

(iv) Given that $X = 2$, find the probability that the digit chosen from set A was 2. [2]

7 The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

(iii) Find the probability that, of 8 randomly chosen children, at least 2 will be able to jump a height of more than 135 cm. [3]

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7 Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X.

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $\text{Var}(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

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1 The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

x	1	4	5	7	9
$P(X = x)$	$4p$	$5p^2$	$1.5p$	$2.5p$	$1.5p$

Find p. [3]

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6 **(i)** State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

(iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

October/November 2010 (63)

2 In a probability distribution the random variable X takes the value x with probability kx, where x takes values 1, 2, 3, 4, 5 only.

(i) Draw up a probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $E(X)$. [2]

May/June 2011 (61)

1 Biscuits are sold in packets of 18. There is a constant probability that any biscuit is broken, independently of other biscuits. The mean number of broken biscuits in a packet has been found to be 2.7. Find the probability that a packet contains between 2 and 4 (inclusive) broken biscuits. [4]

3 The possible values of the random variable X are the 8 integers in the set $\{-2, -1, 0, 1, 2, 3, 4, 5\}$. The probability of X being 0 is $\frac{1}{10}$. The probabilities for all the other values of X are equal. Calculate

(i) $P(X < 2)$, [2]

(ii) the variance of X, [3]

(iii) the value of a for which $P(-a \leqslant X \leqslant 2a) = \frac{17}{35}$. [1]

7 (a) **(i)** Find the probability of getting at least one 3 when 9 fair dice are thrown. [2]

(ii) When n fair dice are thrown, the probability of getting at least one 3 is greater than 0.9. Find the smallest possible value of n. [4]

May/June 2011 (62)

1 A biased die was thrown 20 times and the number of 5s was noted. This experiment was repeated many times and the average number of 5s was found to be 4.8. Find the probability that in the next 20 throws the number of 5s will be less than three. [4]

7 Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the nth choice of cards. [2]

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6 The probability that Sue completes a Sudoku puzzle correctly is 0.75.

(i) Sue attempts n Sudoku puzzles. Find the least value of n for which the probability that she completes all n puzzles correctly is less than 0.06. [3]

Sue attempts 14 Sudoku puzzles every month. The number that she completes successfully is denoted by X.

(ii) Find the value of X that has the highest probability. You may assume that this value is one of the two values closest to the mean of X. [3]

(iii) Find the probability that in exactly 3 of the next 5 months Sue completes more than 11 Sudoku puzzles correctly. [5]

October/November 2011 (61)

3 A team of 4 is to be randomly chosen from 3 boys and 5 girls. The random variable X is the number of girls in the team.

(i) Draw up a probability distribution table for X. [4]

(ii) Given that $E(X) = \frac{5}{2}$, calculate $Var(X)$. [2]

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6 There are a large number of students in Luttley College. 60% of the students are boys. Students can choose exactly one of Games, Drama or Music on Friday afternoons. It is found that 75% of the boys choose Games, 10% of the boys choose Drama and the remainder of the boys choose Music. Of the girls, 30% choose Games, 55% choose Drama and the remainder choose Music.

(i) 6 boys are chosen at random. Find the probability that fewer than 3 of them choose Music. [3]

(ii) 5 Drama students are chosen at random. Find the probability that at least 1 of them is a boy. [6]

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3 A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

6 Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or −. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A−, 2% are B−, 1% are AB− and 7% are O−.

(i) A random sample of 9 people in the UK who are Rhesus + is taken. Find the probability that fewer than 3 are group O+. [6]

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3 A spinner has 5 sides, numbered 1, 2, 3, 4 and 5. When the spinner is spun, the score is the number of the side on which it lands. The score is denoted by the random variable X, which has the probability distribution shown in the table.

x	1	2	3	4	5
$P(X = x)$	0.3	0.15	$3p$	$2p$	0.05

(i) Find the value of p. [1]

A second spinner has 3 sides, numbered 1, 2 and 3. The score when this spinner is spun is denoted by the random variable Y. It is given that $P(Y = 1) = 0.3$, $P(Y = 2) = 0.5$ and $P(Y = 3) = 0.2$.

(ii) Find the probability that, when both spinners are spun together,

(a) the sum of the scores is 4, [3]

(b) the product of the scores is less than 8. [3]

4 In a certain mountainous region in winter, the probability of more than 20 cm of snow falling on any particular day is 0.21.

(i) Find the probability that, in any 7-day period in winter, fewer than 5 days have more than 20 cm of snow falling. [3]

(ii) For 4 randomly chosen 7-day periods in winter, find the probability that exactly 3 of these periods will have at least 1 day with more than 20 cm of snow falling. [4]

May/June 2012 (62)

2 The random variable X has the probability distribution shown in the table.

x	2	4	6
$P(X = x)$	0.5	0.4	0.1

Two independent values of X are chosen at random. The random variable Y takes the value 0 if the two values of X are the same. Otherwise the value of Y is the larger value of X minus the smaller value of X.

(i) Draw up the probability distribution table for Y. [4]

(ii) Find the expected value of Y. [1]

3 In Restaurant Bijoux 13% of customers rated the food as 'poor', 22% of customers rated the food as 'satisfactory' and 65% rated it as 'good'. A random sample of 12 customers who went for a meal at Restaurant Bijoux was taken.

(i) Find the probability that more than 2 and fewer than 12 of them rated the food as 'good'. [3]

On a separate occasion, a random sample of n customers who went for a meal at the restaurant was taken.

(ii) Find the smallest value of n for which the probability that at least 1 person will rate the food as 'poor' is greater than 0.95. [3]

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4 The six faces of a fair die are numbered 1, 1, 1, 2, 3, 3. The score for a throw of the die, denoted by the random variable W, is the number on the top face after the die has landed.

(i) Find the mean and standard deviation of W. [3]

(ii) The die is thrown twice and the random variable X is the sum of the two scores. Draw up a probability distribution table for X. [4]

(iii) The die is thrown n times. The random variable Y is the number of times that the score is 3. Given that $E(Y) = 8$, find $Var(Y)$. [3]

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October/November 2012 (61)

1 Ashok has 3 green pens and 7 red pens. His friend Rod takes 3 of these pens at random, without replacement. Draw up a probability distribution table for the number of green pens Rod takes. [4]

5 A company set up a display consisting of 20 fireworks. For each firework, the probability that it fails to work is 0.05, independently of other fireworks.

(i) Find the probability that more than 1 firework fails to work. [3]

The 20 fireworks cost the company \$24 each. 450 people pay the company \$10 each to watch the display. If more than 1 firework fails to work they get their money back.

(ii) Calculate the expected profit for the company. [4]

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6 A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable X is the sum of the three scores.

(i) Show that $P(X = 9) = \frac{10}{64}$. [3]

(ii) Copy and complete the probability distribution table for X. [3]

x	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{64}$	$\frac{3}{64}$			$\frac{12}{64}$					

(iii) Event R is 'the sum of the three scores is 9'. Event S is 'the product of the three scores is 16'. Determine whether events R and S are independent, showing your working. [5]

October/November 2012 (63)

2 The discrete random variable X has the following probability distribution.

x	-3	0	2	4
$P(X = x)$	p	q	r	0.4

Given that $E(X) = 2.3$ and $Var(X) = 3.01$, find the values of p, q and r. [6]

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5 Fiona uses her calculator to produce 12 random integers between 7 and 21 inclusive. The random variable X is the number of these 12 integers which are multiples of 5.

(i) State the distribution of X and give its parameters. [3]

(ii) Calculate the probability that X is between 3 and 5 inclusive. [3]

Fiona now produces n random integers between 7 and 21 inclusive.

(iii) Find the least possible value of n if the probability that none of these integers is a multiple of 5 is less than 0.01. [3]

May/June 2013 (62)

4 Robert uses his calculator to generate 5 random integers between 1 and 9 inclusive.

(i) Find the probability that at least 2 of the 5 integers are less than or equal to 4. [3]

Robert now generates n random integers between 1 and 9 inclusive. The random variable X is the number of these n integers which are less than or equal to a certain integer k between 1 and 9 inclusive. It is given that the mean of X is 96 and the variance of X is 32.

(ii) Find the values of n and k. [4]

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7 Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

(i) Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag. [2]

(ii) Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate. [4]

(iii) The random variable X is the number of times a chocolate is taken. State the possible values of X and draw up a table to show the probability distribution of X. [5]

May/June 2013 (63)

2 The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

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7 James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable X is defined as follows.

- If the coin shows a **head** then X is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then X is the score on the **first throw** of the die only.

(i) Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

(ii) Show that $P(X = 3) = \frac{3}{16}$. [4]

(iii) Copy and complete the probability distribution table for X. [3]

x	1	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{8}$		$\frac{1}{16}$	$\frac{1}{32}$

Event Q is 'James throws a tail'. Event R is 'the value of X is 7'.

(iv) Determine whether events Q and R are exclusive. Justify your answer. [2]

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7 Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) $= \frac{1}{2}$. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event R is 'the sum of the numbers on the three cards is 11'. Event S is 'the number on the first card taken is a 3'.

(iii) Determine whether events R and S are independent. Justify your answer. [3]

(iv) Determine whether events R and S are exclusive. Justify your answer. [1]

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3 In a large consignment of mangoes, 15% of mangoes are classified as small, 70% as medium and 15% as large.

(i) Yue-chen picks 14 mangoes at random. Find the probability that fewer than 12 of them are medium or large. [3]

(ii) Yue-chen picks n mangoes at random. The probability that none of these n mangoes is small is at least 0.1. Find the largest possible value of n. [3]

7 Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

(i) Find the probability that the two digits chosen are equal. [3]

(ii) Find the probability that one digit is a 5 and one digit is not a 5. [3]

(iii) Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

(iv) The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

May/June 2014 (61)

3 (i) State three conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

George wants to invest some of his monthly salary. He invests a certain amount of this every month for 18 months. For each month there is a probability of 0.25 that he will buy shares in a large company, there is a probability of 0.15 that he will buy shares in a small company and there is a probability of 0.6 that he will invest in a savings account.

(ii) Find the probability that George will buy shares in a small company in at least 3 of these 18 months. [3]

4 A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable X represents the number of paperback books she chooses.

(i) Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

(ii) Draw up the probability distribution table for X. [3]

(iii) You are given that $E(X) = 3$. Find $Var(X)$. [2]

May/June 2014 (62)

1 In a certain country 12% of houses have solar heating. 19 houses are chosen at random. Find the probability that fewer than 4 houses have solar heating. [4]

4 Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

(i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

(ii) Draw up the probability distribution table for the number of heads obtained. [4]

(iii) Find the expectation of the number of heads obtained. [2]

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3 A pet shop has 6 rabbits and 3 hamsters. 5 of these pets are chosen at random. The random variable X represents the number of hamsters chosen.

(i) Show that the probability that exactly 2 hamsters are chosen is $\frac{10}{21}$. [2]

(ii) Draw up the probability distribution table for X. [4]

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2 The number of phone calls, X, received per day by Sarah has the following probability distribution.

x	0	1	2	3	4	$\geqslant 5$
$P(X = x)$	0.24	0.35	$2k$	k	0.05	0

(i) Find the value of k. [2]

(ii) Find the mode of X. [1]

(iii) Find the probability that the number of phone calls received by Sarah on any particular day is more than the mean number of phone calls received per day. [3]

5 Screws are sold in packets of 15. Faulty screws occur randomly. A large number of packets are tested for faulty screws and the mean number of faulty screws per packet is found to be 1.2.

(i) Show that the variance of the number of faulty screws in a packet is 1.104. [2]

(ii) Find the probability that a packet contains at most 2 faulty screws. [3]

Damien buys 8 packets of screws at random.

(iii) Find the probability that there are exactly 7 packets in which there is at least 1 faulty screw. [4]

October/November 2014 (62)

3 **(i)** Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown. Find the probability that the numbers shown on the four dice add up to 5. [3]

(ii) Four fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown on 7 occasions. Find the probability that the numbers shown on the four dice add up to 5 on exactly 1 or 2 of the 7 occasions. [4]

4 Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

(i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]

(ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find $E(X)$. [4]

October/November 2014 (63)

3 The number of books read by members of a book club each year has the binomial distribution $B(12, 0.7)$.

(i) State the greatest number of books that could be read by a member of the book club in a particular year and find the probability that a member reads this number of books. [2]

(ii) Find the probability that a member reads fewer than 10 books in a particular year. [3]

7 A box contains 2 green apples and 2 red apples. Apples are taken from the box, one at a time, without replacement. When both red apples have been taken, the process stops. The random variable X is the number of apples which have been taken when the process stops.

(i) Show that $P(X = 3) = \frac{1}{3}$. [3]

(ii) Draw up the probability distribution table for X. [3]

Another box contains 2 yellow peppers and 5 orange peppers. Three peppers are taken at random from the box without replacement.

(iii) Given that at least 2 of the peppers taken from the box are orange, find the probability that all 3 peppers are orange. [5]

May/June 2015 (61)

6 (i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

May/June 2015 (62)

1 A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

5 A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

May/June 2015 (63)

4 A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

October/November 2015 (61)

1 In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

6 Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

<u>October/November 2015 (62)</u>

6 A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	−3	−2			
	−2			1	
	−1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find $\text{Var}(X)$. [3]

(iv) Find the probability that X is even, given that X is positive. [2]

October/November 2015 (63)

7 A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

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