The Normal Distribution
(Past Year Topical Questions 2010-2015)

<u>May/June 2010 (61)</u>

3 The random variable X is the length of time in minutes that Jannon takes to mend a bicycle puncture. X has a normal distribution with mean μ and variance σ^2. It is given that $P(X > 30.0) = 0.1480$ and $P(X > 20.9) = 0.6228$. Find μ and σ. [5]

5 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

   (i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

   (ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

   (iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

<u>May/June 2010 (62)</u>

2 The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

   (i) Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

   (ii) Find the probability that, in a random sample of 6 pencils, at least two have lengths less than 10.9 cm. [3]

4 The random variable X is normally distributed with mean μ and standard deviation σ.

   (i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]

   (ii) With a different relationship between μ and σ, it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ. [3]

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7   The heights that children of a particular age can jump have a normal distribution. On average, 8 children out of 10 can jump a height of more than 127 cm, and 1 child out of 3 can jump a height of more than 135 cm.

(i)   Find the mean and standard deviation of the heights the children can jump. [5]

(ii) Find the probability that a randomly chosen child will **not** be able to jump a height of 145 cm. [3]

October/November 2010 (61)

2   On average, 2 apples out of 15 are classified as being underweight. Find the probability that in a random sample of 200 apples, the number of apples which are underweight is more than 21 and less than 35. [5]

3   The times taken by students to get up in the morning can be modelled by a normal distribution with mean 26.4 minutes and standard deviation 3.7 minutes.

(i)   For a random sample of 350 students, find the number who would be expected to take longer than 20 minutes to get up in the morning. [3]

(ii) 'Very slow' students are students whose time to get up is more than 1.645 standard deviations above the mean. Find the probability that fewer than 3 students from a random sample of 8 students are 'very slow'. [4]

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5   The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.

(i)   For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]

(ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d. [4]

October/November 2010 (63)

1   Name the distribution and suggest suitable numerical parameters that you could use to model the weights in kilograms of female 18-year-old students. [2]

7   The times spent by people visiting a certain dentist are independent and normally distributed with a mean of 8.2 minutes. 79% of people who visit this dentist have visits lasting less than 10 minutes.

   (i) Find the standard deviation of the times spent by people visiting this dentist. [3]

   (ii) Find the probability that the time spent visiting this dentist by a randomly chosen person deviates from the mean by more than 1 minute. [3]

   (iii) Find the probability that, of 6 randomly chosen people, more than 2 have visits lasting longer than 10 minutes. [3]

   (iv) Find the probability that, of 35 randomly chosen people, fewer than 16 have visits lasting less than 8.2 minutes. [5]

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5   (a) The random variable X is normally distributed with mean μ and standard deviation σ. It is given that $3\mu = 7\sigma^2$ and that $P(X > 2\mu) = 0.1016$. Find μ and σ. [4]

   (b) It is given that $Y \sim N(33, 21)$. Find the value of a given that $P(33 - a < Y < 33 + a) = 0.5$. [4]

May/June 2011 (62)

2   In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

   (i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]

   (ii) Give a reason why the use of a normal approximation is justified. [1]

6 The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

    (i) Find the value of μ.   [3]

    (ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm.   [6]

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5 The random variable X is normally distributed with mean μ and standard deviation $\frac{1}{4}\mu$. It is given that $P(X > 20) = 0.04$.

    (i) Find μ.   [3]

    (ii) Find $P(10 < X < 20)$.   [3]

    (iii) 250 independent observations of X are taken. Find the probability that at least 235 of them are less than 20.   [5]

October/November 2011 (61)

1 When a butternut squash seed is sown the probability that it will germinate is 0.86, independently of any other seeds. A market gardener sows 250 of these seeds. Use a suitable approximation to find the probability that more than 210 germinate.   [5]

5 The weights of letters posted by a certain business are normally distributed with mean 20 g. It is found that the weights of 94% of the letters are within 12 g of the mean.

    (i) Find the standard deviation of the weights of the letters.   [3]

    (ii) Find the probability that a randomly chosen letter weighs more than 13 g.   [3]

    (iii) Find the probability that at least 2 of a random sample of 7 letters have weights which are more than 12 g above the mean.   [3]

5 A triangular spinner has one red side, one blue side and one green side. The red side is weighted so that the spinner is four times more likely to land on the red side than on the blue side. The green side is weighted so that the spinner is three times more likely to land on the green side than on the blue side.

(i) Show that the probability that the spinner lands on the blue side is $\frac{1}{8}$. [1]

(iii) The spinner is spun 136 times. Use a suitable approximation to find the probability that it lands on the blue side fewer than 20 times. [5]

7 (i) In a certain country, the daily minimum temperature, in °C, in winter has the distribution N(8, 24). Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7 °C and 12 °C. [3]

The daily minimum temperature, in °C, in another country in winter has a normal distribution with mean μ and standard deviation 2μ.

(ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]

(iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]

(iv) The probability of the minimum temperature being above 6 °C on any winter day is 0.0735. Find the value of μ. [3]

1 The random variable X is normally distributed and is such that the mean μ is three times the standard deviation σ. It is given that P($X < 25$) = 0.648.

(i) Find the values of μ and σ. [4]

(ii) Find the probability that, from 6 random values of X, exactly 4 are greater than 25. [2]

5

6 Human blood groups are identified by two parts. The first part is A, B, AB or O and the second part (the Rhesus part) is + or −. In the UK, 35% of the population are group A+, 8% are B+, 3% are AB+, 37% are O+, 7% are A−, 2% are B−, 1% are AB− and 7% are O−.

   (ii) A random sample of 150 people in the UK is taken. Find the probability that more than 60 people are group A+. [5]

May/June 2012 (61)

1 It is given that $X \sim N(28.3, 4.5)$. Find the probability that a randomly chosen value of X lies between 25 and 30. [3]

6 The lengths of body feathers of a particular species of bird are modelled by a normal distribution. A researcher measures the lengths of a random sample of 600 body feathers from birds of this species and finds that 63 are less than 6 cm long and 155 are more than 12 cm long.

   (i) Find estimates of the mean and standard deviation of the lengths of body feathers of birds of this species. [5]

   (ii) In a random sample of 1000 body feathers from birds of this species, how many would the researcher expect to find with lengths more than 1 standard deviation from the mean? [4]

May/June 2012 (62)

7 The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

   (i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

   (ii) Find the mean and standard deviation of the times to play this symphony. [5]

   (iii) Assuming that the times to play the two symphonies are independent of each other, find the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

May/June 2012 (63)

6 The lengths, in cm, of trout in a fish farm are normally distributed. 96% of the lengths are less than 34.1 cm and 70% of the lengths are more than 26.7 cm.

(i) Find the mean and the standard deviation of the lengths of the trout. [5]

In another fish farm, the lengths of salmon, X cm, are normally distributed with mean 32.9 cm and standard deviation 2.4 cm.

(ii) Find the probability that a randomly chosen salmon is 34 cm long, correct to the nearest centimetre. [3]

(iii) Find the value of t such that $P(31.8 < X < t) = 0.5$. [4]

October/November 2012 (61)

3 Lengths of rolls of parcel tape have a normal distribution with mean 75 m, and 15% of the rolls have lengths less than 73 m.

(i) Find the standard deviation of the lengths. [3]

Alison buys 8 rolls of parcel tape.

(ii) Find the probability that fewer than 3 of these rolls have lengths more than 77 m. [3]

6 Ana meets her friends once every day. For each day the probability that she is early is 0.05 and the probability that she is late is 0.75. Otherwise she is on time.

(i) Find the probability that she is on time on fewer than 20 of the next 96 days. [5]

(ii) If she is early there is a probability of 0.7 that she will eat a banana. If she is late she does not eat a banana. If she is on time there is a probability of 0.4 that she will eat a banana. Given that for one particular meeting with friends she does not eat a banana, find the probability that she is on time. [4]

<u>October/November 2012 (62)</u>

2   The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

(i) Find the probability that, on a randomly chosen day, the company makes a profit between $10 000 and $12 000.   [3]

(ii) Find the probability that the company makes a loss on exactly 1 of the next 4 consecutive days.   [4]

4   The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

(i) Find the mean and standard deviation.   [4]

(ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5.   [5]

<u>October/November 2012 (63)</u>

1   In a normal distribution with mean 9.3, the probability of a randomly chosen value being greater than 5.6 is 0.85. Find the standard deviation.   [3]

5   The random variable X is such that $X \sim N(82, 126)$.

(i) A value of X is chosen at random and rounded to the nearest whole number. Find the probability that this whole number is 84.   [3]

(ii) Five independent observations of X are taken. Find the probability that at most one of them is greater than 87.   [4]

(iii) Find the value of k such that $P(87 < X < k) = 0.3$.   [5]

<u>May/June 2013 (61)</u>

2   Assume that, for a randomly chosen person, their next birthday is equally likely to occur on any day of the week, independently of any other person's birthday. Find the probability that, out of 350 randomly chosen people, at least 47 will have their next birthday on a Monday.   [5]

4 **(a)** The random variable Y is normally distributed with positive mean μ and standard deviation $\frac{1}{2}\mu$. Find the probability that a randomly chosen value of Y is negative. [3]

(b) The weights of bags of rice are normally distributed with mean 2.04 kg and standard deviation σ kg. In a random sample of 8000 such bags, 253 weighed over 2.1 kg. Find the value of σ. [4]

May/June 2013 (62)

1 The random variable Y is normally distributed with mean equal to five times the standard deviation. It is given that $P(Y > 20) = 0.0732$. Find the mean. [3]

3 Cans of lemon juice are supposed to contain 440 ml of juice. It is found that the actual volume of juice in a can is normally distributed with mean 445 ml and standard deviation 3.6 ml.

(i) Find the probability that a randomly chosen can contains less than 440 ml of juice. [3]

It is found that 94% of the cans contain between $(445 - c)$ ml and $(445 + c)$ ml of juice.

(ii) Find the value of c. [3]

May/June 2013 (63)

3 Buildings in a certain city centre are classified by height as tall, medium or short. The heights can be modelled by a normal distribution with mean 50 metres and standard deviation 16 metres. Buildings with a height of more than 70 metres are classified as tall.

(i) Find the probability that a building chosen at random is classified as tall. [2]

(ii) The rest of the buildings are classified as medium and short in such a way that there are twice as many medium buildings as there are short ones. Find the height below which buildings are classified as short. [5]

4 In a certain country, on average one student in five has blue eyes.

(i) For a random selection of n students, the probability that none of the students has blue eyes is less than 0.001. Find the least possible value of n. [3]

(ii) For a random selection of 120 students, find the probability that fewer than 33 have blue eyes. [4]

October/November 2013 (61)

1 It is given that $X \sim N(30, 49)$, $Y \sim N(30, 16)$ and $Z \sim N(50, 16)$. On a single diagram, with the horizontal axis going from 0 to 70, sketch three curves to represent the distributions of X, Y and Z.
[3]

5 Lengths of a certain type of carrot have a normal distribution with mean 14.2 cm and standard deviation 3.6 cm.

    (i) 8% of carrots are shorter than c cm. Find the value of c.
[3]

    (ii) Rebekah picks 7 carrots at random. Find the probability that at least 2 of them have lengths between 15 and 16 cm.
[6]

October/November 2013 (62)

1 It is given that $X \sim N(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than -2.4.
[3]

3 The amount of fibre in a packet of a certain brand of cereal is normally distributed with mean 160 grams. 19% of packets of cereal contain more than 190 grams of fibre.

    (i) Find the standard deviation of the amount of fibre in a packet.
[3]

    (ii) Kate buys 12 packets of cereal. Find the probability that at least 1 of the packets contains more than 190 grams of fibre.
[2]

5 On trains in the morning rush hour, each person is either a student with probability 0.36, or an office worker with probability 0.22, or a shop assistant with probability 0.29 or none of these.

    (i) 8 people on a morning rush hour train are chosen at random. Find the probability that between 4 and 6 inclusive are office workers.
[3]

    (ii) 300 people on a morning rush hour train are chosen at random. Find the probability that between 31 and 49 inclusive are neither students nor office workers nor shop assistants.
[6]

October/November 2013 (63)

2 A factory produces flower pots. The base diameters have a normal distribution with mean 14 cm and standard deviation 0.52 cm. Find the probability that the base diameters of exactly 8 out of 10 randomly chosen flower pots are between 13.6 cm and 14.8 cm. [5]

5 (a) The random variable X is normally distributed with mean 82 and standard deviation 7.4. Find the value of q such that $P(82 - q < X < 82 + q) = 0.44$. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. It is given that $5\mu = 2\sigma^2$ and that $P(Y < \frac{1}{2}\mu) = 0.281$. Find the values of μ and σ. [4]

May/June 2014 (61)

1 The petrol consumption of a certain type of car has a normal distribution with mean 24 kilometres per litre and standard deviation 4.7 kilometres per litre. Find the probability that the petrol consumption of a randomly chosen car of this type is between 21.6 kilometres per litre and 28.7 kilometres per litre. [4]

2 Lengths of a certain type of white radish are normally distributed with mean μ cm and standard deviation σ cm. 4% of these radishes are longer than 12 cm and 32% are longer than 9 cm. Find μ and σ. [5]

May/June 2014 (62)

7 The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% of these days he spends more than 1.35 hours on his homework.

(i) Find the value of σ. [3]

(ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. [2]

(iii) A random sample of 200 days in term-time is taken. Use an approximation to find the probability that the number of days on which Rafa spends more than 1.35 hours on his homework is between 163 and 173 inclusive. [6]

May/June 2014 (63)

2 There is a probability of $\frac{1}{9}$ that Wenjie goes out with her friends on any particular day. 252 days are chosen at random.

   (i) Use a normal approximation to find the probability that the number of days on which Wenjie goes out with her friends is less than than 30 or more than 44. [5]

   (ii) Give a reason why the use of a normal approximation is justified. [1]

5 When Moses makes a phone call, the amount of time that the call takes has a normal distribution with mean 6.5 minutes and standard deviation 1.76 minutes.

   (i) 90% of Moses's phone calls take longer than t minutes. Find the value of t. [3]

   (ii) Find the probability that, in a random sample of 9 phone calls made by Moses, more than 7 take a time which is within 1 standard deviation of the mean. [5]

October/November 2014 (61)

6 A farmer finds that the weights of sheep on his farm have a normal distribution with mean 66.4 kg and standard deviation 5.6 kg.

   (i) 250 sheep are chosen at random. Estimate the number of sheep which have a weight of between 70 kg and 72.5 kg. [5]

   (ii) The proportion of sheep weighing less than 59.2 kg is equal to the proportion weighing more than y kg. Find the value of y. [2]

Another farmer finds that the weights of sheep on his farm have a normal distribution with mean μ kg and standard deviation 4.92 kg. 25% of these sheep weigh more than 67.5 kg.

   (iii) Find the value of μ. [3]

5 (a) The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.

   (i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]

   (ii) Find the value of q such that $P(X < q) = 0.75$. [3]

   (b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$. [3]

7 In Marumbo, three quarters of the adults own a cell phone.

   (i) A random sample of 8 adults from Marumbo is taken. Find the probability that the number of adults who own a cell phone is between 4 and 6 inclusive. [3]

   (ii) A random sample of 160 adults from Marumbo is taken. Use an approximation to find the probability that more than 114 of them own a cell phone. [5]

   (iii) Justify the use of your approximation in part (ii). [1]

October/November 2014 (63)

1 Packets of tea are labelled as containing 250 g. The actual weight of tea in a packet has a normal distribution with mean 260 g and standard deviation σ g. Any packet with a weight less than 250 g is classed as 'underweight'. Given that 1% of packets of tea are underweight, find the value of σ. [3]

5 Gem stones from a certain mine have weights, X grams, which are normally distributed with mean 1.9 g and standard deviation 0.55 g. These gem stones are sorted into three categories for sale depending on their weights, as follows.

   Small: under 1.2 g   Medium: between 1.2 g and 2.5 g   Large: over 2.5 g

   (i) Find the proportion of gem stones in each of these three categories. [5]

   (ii) Find the value of k such that $P(k < X < 2.5) = 0.8$. [4]

13

May/June 2015 (61)

1   The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ.   [4]

6   **(i)** In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer.   [4]

     (ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer.   [5]

     (iii) Justify your use of the approximation in part **(ii)**.   [1]

May/June 2015 (62)

7   **(a)** Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

     (i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run.   [4]

     (ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t.   [3]

     (b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ.   [5]

May/June 2015 (63)

1   The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ.   [3]

3   On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard.   [5]

5   The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as 'large'.

    (i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large.

        [6]

    (ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n.

        [3]

October/November 2015 (61)

2   The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ.   [4]

4   (a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

        Find the mean and standard deviation of Amy's pulse rate.   [4]

    (b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute.   [3]

7   The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

    (i) Find the probability of throwing a 3.   [3]

    (ii) The die is thrown three times. Find the probability of throwing two 5s and one 4.   [3]

    (iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times.   [5]

October/November 2015 (62)

7   (a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

        (i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

      The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

        (ii) Find the value of m. [3]

        (iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

    (b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

October/November 2015 (63)

4   The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

    (i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

    (ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

7   A factory makes water pistols, 8% of which do not work properly.

    (i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

    (ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

    (iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

    (iv) Justify the use of your approximation in part (iii). [1]