Indices and surds

(Past Year Topical Questions 2012-2017)

May/June 2012 (12)

6.

(i) Express $\frac{8}{\sqrt{3}+1}$ in the form $a(\sqrt{3}-1)$, where a is an integer. [2]

An equilateral triangle has sides of length $\frac{8}{\sqrt{3}+1}$.

(ii) Show that the height of the triangle is $6-2\sqrt{3}$. [2]

(iii) Hence, or otherwise, find the area of the triangle in the form $p\sqrt{3}-q$, where p and q are integers. [2]

Oct/Nov 2012 (11)

7.

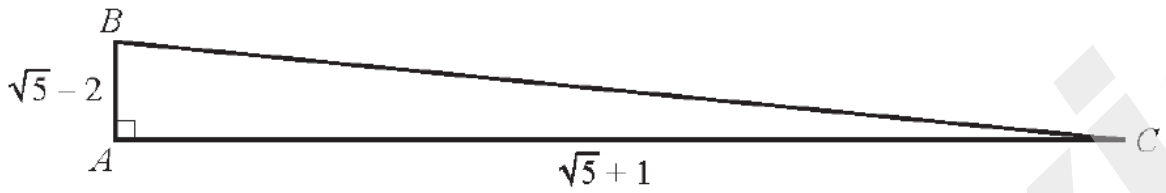
(a) **(i)** Show that $3\sqrt{5}-2\sqrt{2}$ is a square root of $53-12\sqrt{10}$. [1]

(ii) State the other square root of $53-12\sqrt{10}$. [1]

(b) Express $\frac{6\sqrt{3}+7\sqrt{2}}{4\sqrt{3}+5\sqrt{2}}$ in the form $a+b\sqrt{6}$, where a and b are integers to be found. [4]

May/June 2013 (11)

7.



The diagram shows a triangle ABC in which angle $A = 90°$. Sides AB and AC are $\sqrt{5} - 2$ and $\sqrt{5} + 1$ respectively. Find

(i) tan B in the form $a + b\sqrt{5}$, where a and b are integers. [3]

(ii) $\sec^2 B$ in the form $c + d\sqrt{5}$, where c and d are integers. [4]

May/June 2014 (12)

5.

(i) Given that $2^{5x} \times 4^y = \frac{1}{8}$, show that $5x + 2y = -3$. [3]

(ii) Solve the simultaneous equations $2^{5x} \times 4^y = \frac{1}{8}$ and $7^x \times 49^{2y} = 1$. [4]

May/June 2014 (13)

2.

Given that $2^{4x} \times 4^{y} \times 8^{x-y} = 1$ and $3^{x+y} = \frac{1}{3}$, find the value of x and of y. [4]

Oct/Nov 2014 (11)

4.

(i) Using the substitution $y = 5^{x}$, show that the equation $5^{2x+1} - 5^{x+1} + 2 = 2(5^{x})$ can be written in the form $ay^2 + by + 2 = 0$, where a and b are constants to be found. [2]

(ii) Hence solve the equation $5^{2x+1} - 5^{x+1} + 2 = 2(5^{x})$. [4]

Oct/Nov 2014 (13)
10.

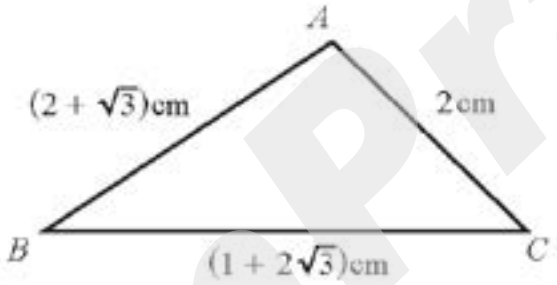
(a) Solve the following simultaneous equations.

$$\frac{5^x}{25^{3y-2}} = 1$$

$$\frac{3^x}{27^{y-1}} = 81$$

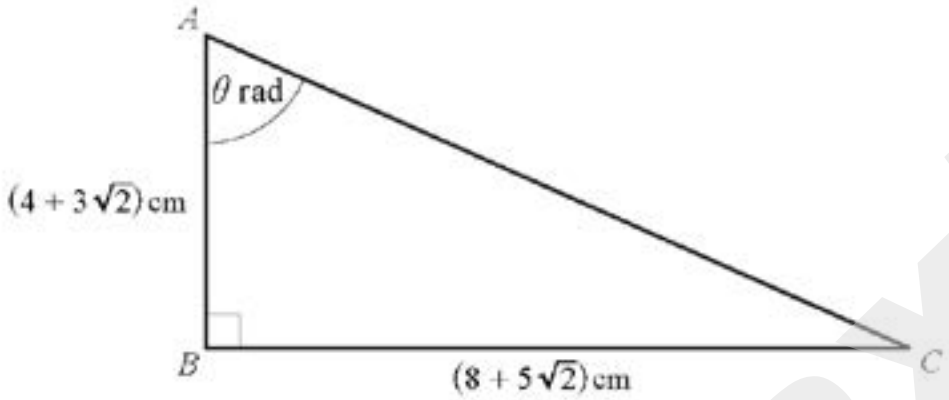
[5]

(b) The diagram shows a triangle ABC such that $AB = (2+\sqrt{3})$ cm, $BC = (1+2\sqrt{3})$ cm and $AC = 2$ cm.



Without using a calculator, find the value of $\cos A$ in the form $a + b\sqrt{3}$, where a and b are constants to be found. [4]

May/June 2015 (11)
2.



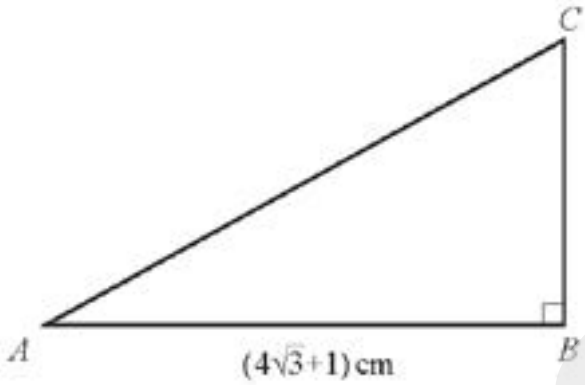
The diagram shows the triangle ABC where angle B is a right angle, $AB = (4 + 3\sqrt{2})$ cm, $BC = (8 + 5\sqrt{2})$ cm and angle $BAC = \theta$ radians. Showing all your working, find

(i) $\tan\theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [2]

(ii) $\sec^2\theta$ in the form $c + d\sqrt{2}$, where c and d are integers. [3]

Oct/Nov 2015 (13)

4.



The diagram shows triangle ABC with side $AB = (4\sqrt{3} + 1)$ cm. Angle B is a right angle. It is given that the area of this triangle is $\frac{47}{2}$ cm^2.

(i) Find the length of the side BC in the form $(a\sqrt{3} + b)$ cm, where a and b are integers. [3]

(ii) Hence find the length of the side AC in the form $p\sqrt{2}$ cm, where p is an integer. [2]

May/June 2016 (11)

2.

(a) Solve the equation $16^{3x-1} = 8^{x+2}$. [3]

(b) Given that $\frac{\left(a^{\frac{1}{3}}b^{-\frac{1}{2}}\right)^3}{a^{-\frac{2}{3}}b^{\frac{1}{2}}} = a^p b^q$, find the value of each of the constants p and q. [2]

May/June 2016 (12)

4.

Find the positive value of x for which $(4+\sqrt{5})x^2 + (2-\sqrt{5})x - 1 = 0$, giving your answer in the form $\frac{a+\sqrt{5}}{b}$, where a and b are integers. [6]

Oct/Nov 2016(11)

2.

Given that $\frac{p^{\frac{1}{3}}q^{-\frac{1}{2}}r^{\frac{3}{2}}}{p^{-\frac{2}{3}}\sqrt{(qr)^5}} = p^a q^b r^c$, find the value of each of the integers a, b and c. [3]

Oct/Nov 2016 (13)

2

Express $\frac{4m\sqrt{m} - \frac{9}{\sqrt{m}}}{2\sqrt{m} + \frac{3}{\sqrt{m}}}$ in the form $Am + B$, where A and B are integers to be found. [3]

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May/June 2017 (11)

3.

(a) Simplify $\sqrt{x^8y^{10}} \div \sqrt[3]{x^3y^{-6}}$, giving your answer in the form $x^a y^b$, where a and b are integers. [2]

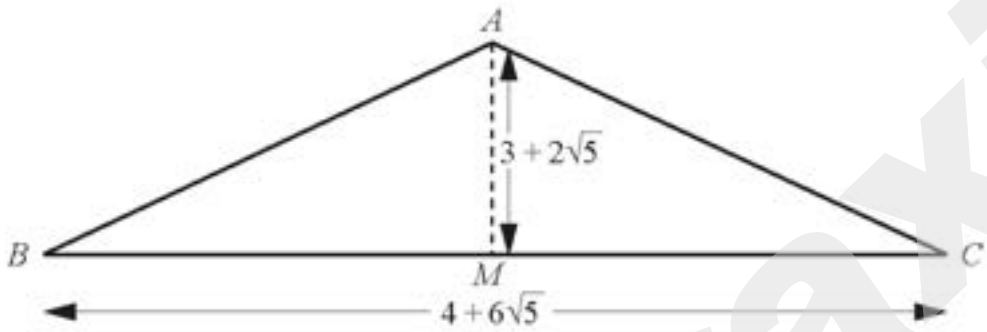
(b) **(i)** Show that $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}}$ can be written in the form $(t-2)^p(qt+r)$, where p, q and r are constants to be found. [3]

(ii) Hence solve the equation $4(t-2)^{\frac{1}{2}} + 5(t-2)^{\frac{3}{2}} = 0$. [1]

May/June 2017 (13)

4.

In this question, all dimensions are in centimetres.



The diagram shows an isosceles triangle ABC, where $AB = AC$. The point M is the mid-point of BC. Given that $AM = 3 + 2\sqrt{5}$ and $BC = 4 + 6\sqrt{5}$, find, **without using a calculator,**

(i) the area of triangle ABC, [2]

(ii) $\tan ABC$, giving your answer in the form $\frac{a + b\sqrt{5}}{c}$ where a, b and c are positive integers. [3]

Oct/Nov 2017 (11)

3.

(a) Given that $T = 2\pi l^{\frac{1}{2}} g^{-\frac{1}{2}}$, express l in terms of T, g and π. [2]

(b) By using the substitution $y = x^{\frac{1}{3}}$, or otherwise, solve $x^{\frac{2}{3}} - 4x^{\frac{1}{3}} + 3 = 0$. [4]